

Math 2280-001
Week 14 concepts and homework, due April 24

7.3: translation and partial fractions for Laplace transform IVPs

week 14.1. Use Laplace transforms to solve the system of DE's that arose from an input-output model on the second midterm, that you've also already solved using matrix exponentials:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 120 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hint: If you take the Laplace transform of the two equations you will get a system of two equations for

$$\underline{X}(s) = \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

which you can solve. After some partial fractions decomposition you will be able to find the inverse Laplace transforms $x_1(t), x_2(t)$. It may be helpful to use the matrix vector form of the differential equation to find $X(s)$ since if

$$\underline{x}'(t) = A \underline{x}(t) + \underline{f}(t)$$

then it follows after transforming each equation that

$$s \underline{X}(s) - \underline{x}_0 = A \underline{X}(s) + \underline{F}(s)$$

i.e.

$$(sI - A)\underline{X}(s) = \underline{x}_0 + \underline{F}(s).$$

7.4 convolutions (discussed in class on Friday April 17).

2, 13, 37.

7.5 using the unit step function to turn forcing on and off (discussed in class on Friday April 17).

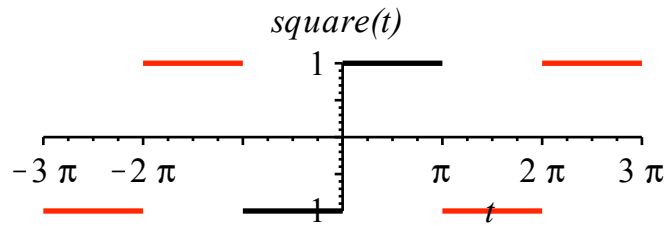
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9.1 Fourier series for 2π -periodic functions

3, 10, 30

week 14.2 By computing Fourier coefficients, verify that the Fourier series for $\text{square}(t)$, the 2π -periodic square wave extension of

$$f(t) = \begin{cases} -1 & -\pi < t < 0 \\ 1 & 0 < t < \pi \end{cases}$$



is

$$\text{square}(t) \sim \frac{4}{\pi} \sum_{n \text{ odd}} \frac{1}{n} \sin(n t).$$

9.2 General Fourier series for $2L$ -periodic functions

2, 2

week 14.3 Notice that $\text{square}(t)$ in week 14.2 is related to the $f(u)$ in 9.2.2. In fact,

$$f(u) = \frac{1}{2} + \frac{1}{2} \text{square}\left(\frac{\pi}{5}u\right).$$

a) Use this fact to re-find the answer to 4.2, by using the Fourier series for $\text{square}(t)$.

b) Use technology to graph the partial sum of the first 10 non-zero terms in your Fourier sine series of $f(t)$ in 9.2.2 to verify that it is close to the graph of the square wave.

week 14.4 Use technology to check your Fourier coefficients in 9.2.2.

9.3 Fourier sine and cosine series

17, 19, 20 **Postpone 19, 20 until next assignment**

week 14.5 Recall that an even function $f(t)$ is one for which $f(-t) = f(t)$ always hold, and that an odd function $g(t)$ is one for which $g(-t) = -g(t)$ always holds.

a) Let $f_1(t), f_2(t)$ be even. Prove $f_1(t)f_2(t)$ is also even.

b) Let $f(t)$ even, and $g(t)$ odd. Prove $f(t)g(t)$ is odd.

c) Let $g_1(t), g_2(t)$ be odd. Prove that $g_1(t)g_2(t)$ is even.

Postpone until next assignment:

week 14.6) In 9.3.19 you successively antidifferentiate the sawtooth Fourier series (for $f(t) = t$ on

$[-\pi, \pi]$ extended to be 2π -periodic) three times to find that the 2π -periodic extension of $f(t) = \frac{t^4}{24}$

has Fourier series

$$\frac{t^4}{24} = \frac{\pi^2 t^2}{12} - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \cos(n t) + 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}.$$

(We'll do the first integration in class on Wednesday.)

Compare your answer by having Maple find the Fourier coefficients of $\frac{t^4}{24}$ directly. Hint: The Maple

commands you would need are on page 3 of Monday April 20 class notes. The two answers will not immediately look identical, although they are equivalent.

