Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in. When you are asked to check answers with technology, please hand in a screen shot or output from the check. You can do all of the technology checks together and put them at the end of your assignment. Make sure to indicate clearly which checks correspond to which problems.

7.1: Laplace transforms and inverse transforms.
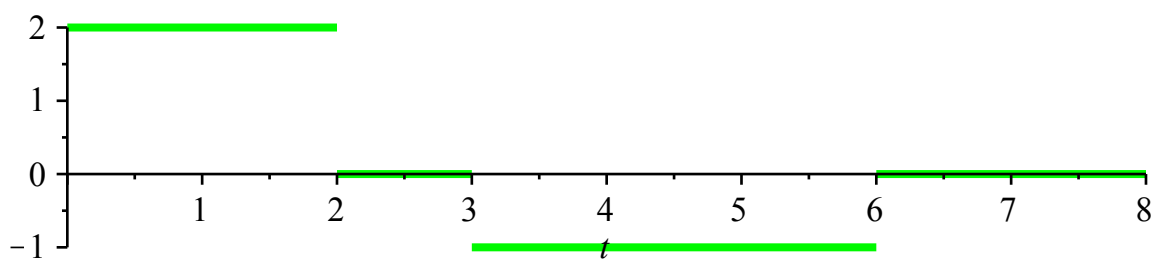*Use the definition of Laplace transform and integration techniques to compute Laplace transforms of* $f(t)$:
1, 3, 7, 9
week 13.1) Use the definition of Laplace transform

$$\mathcal{L}\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}\, dt$$

and integration by parts to compute the Laplace transform $F(s)$ of $f(t) = 4\, t\, e^{2\, t}$.

week 13.2) Consider the function $f(t)$ which is zero for $t > 6$, is piecewise constant, and has this graph

Break the Laplace transform integral from zero to infinity into the sum of integrals over four subintervals (two of the integrals will be zero and use antidifferentiation for the other two), in order to compute the Laplace transform $F(s)$ of $f(t)$.

Use Laplace transform table, linearity (and useful trig identities) to compute Laplace transforms and inverse Laplace transforms.
13, 17, 19, 23, 29
week 13.3a) Use the Laplace transform table in the front cover of the text, algebra, and the linearity of Laplace transform, to compute the Laplace transform $F(s)$ for

$$f(t) = 2\, e^{-t}\sin(3\, t) - 2\, e^{-3\, t}(2 - 3\, t) - 4\, t^2.$$

b) Check your answer with technology. (Maple commands are shown at the end of this document, but you can use whatever software is most convenient.)

week 13.4a) Use the Laplace transform table and linearity to compute the inverse Laplace transform $g(t)$ of

$$G(s) = \frac{3}{s+4} + \frac{1}{(s-4)^2} - \frac{2\, s + 6}{s^2 + 4} + \frac{2\, s + 3}{\left(s^2 + 9\right)^2}\, .$$

b) Check your answer with technology. (See below for Maple commands.)

7.2: Transforming and solving initial value problems via Laplace transforms;
3, 7, 9,19.

week 13.5) Practice completing the square (and "completing the linear" in the numerator) by finding the inverse Laplace transform $f(t)$ for $F(s) = \dfrac{9\,s + 10}{s^2 + 2\,s + 17}$.

week 13.6) Use Laplace transforms to solve the underdamped initial value problem
$$x''(t) + 4\,x'(t) + 13\,x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = 4$$

week 13.7a) Use Laplace transforms (and partial fractions) to solve the initial value problem for $x(t)$:
$$x''(t) + 6\,x'(t) + 9\,x(t) = 30\cos(3\,t)$$
$$x(0) = 0$$
$$x'(0) = 0$$
b) Check your answer with technology.
c) Identify the steady periodic solution, and convert it to amplitude-phase form.

7.2-7.3: Laplace transform table entries; partial fractions to simplify F(s); the translation theorem with completing the square, to identify inverse Laplace transforms; applying these and other techniques to initial value problems.
7.2: **20**
7.3: 3, 7, 9, 17, **20**, 27, **30**, **32**, 34.

week 13.8) With access to a Laplace transform table it is possible to very quickly recover the general solutions to key mechanical oscillation problems (some of which are real messy to derive with Chapter 3 techniques). Do this for
a) undamped forced oscillation, $\omega \neq \omega_0$:

$$x''(t) + \omega_0^2\,x(t) = \frac{F_0}{m}\cos(\omega\,t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

b) undamped forced oscillation, $\omega = \omega_0$:

$$x''(t) + \omega_0^2\,x(t) = \frac{F_0}{m}\cos(\omega_0\,t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

Notes: Here are Maple commands to check partial fractions and to compute Laplace transforms and inverse Laplace transforms:
```
> with(inttrans) :  # to see the integral transform list in this library replace : with ;
> f1 := t→t·exp(3·t)·cos(4·t);
    laplace(f1(t), t, s);  # for more info on this command use help windows
```

$$f1 := t \rightarrow t\,e^{3\,t}\cos(4\,t)$$

$$\frac{s^2 - 6\,s - 7}{\left((s-3)^2 + 16\right)^2} \tag{1}$$

> $F1 := s \rightarrow \dfrac{s^2 - 6\,s - 7}{\left((s-3)^2 + 16\right)^2};$
> $invlaplace(F1(s), s, t);$

$$F1 := s \rightarrow \frac{s^2 - 6\,s - 7}{\left((s-3)^2 + 16\right)^2}$$

$$t\,e^{3\,t}\cos(4\,t) \tag{2}$$

> $convert(F1(s), parfrac, s);$

$$\frac{1}{s^2 - 6\,s + 25} - \frac{32}{\left(s^2 - 6\,s + 25\right)^2} \tag{3}$$

>