Math 2280-001

Monday, April 27

Course review

Final exam: Wednesday May 6, 8:00 a.m. -10:00 a.m. (I will let you work until 10:15). This is the official University time and location - our lecture room LCB 215 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. You will be provided a Laplace Transform table and the formulas for Fourier coefficients. The algebra and math on the exam should all be doable by hand.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters.

Chapters:

1-2: 10-20% first order DEs

3: 15-30% linear differential equations and applications

4.1, 5: 30-50% linear systems of differential equations and applications

7: 15-25% Laplace transforms and applications

9.1-9.4: 10-15% Fourier series and applications to forced oscillations

On the next page is a more detailed list of the topics we’ve investigated this semester. They are more inter-related than you may have realized at the time, so let’s discuss the connections. Then we’ll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

slope fields, Euler approximation

phase diagrams for autonomous DEs

equilibrium solutions

stability

existence-uniqueness thm for IVPs

methods:

separable

linear

applications

populations

velocity-acceleration models

input-output models

3 Linear differential equations

IVP existence and uniqueness

Linear DEs

Homogeneous solution space,

its dimension, and why

superposition, x(t)= xP+ xH

linear transformations

aka superposition

fundamental theorem for solution

space to L(y)=f when L is linear

(We use ector space concepts:

vector spaces and subspaces

linear combinations

linear dependence/independence

span

basis and dimension)

Constant coefficient linear DEs

xH  via characteristic polynomial

Euler’s formula, complex roots

xP  via undetermined coefficients

solving IVPs

applications:

mechanical configurations

unforced: undamped and damped

cos and sin addition angle formulas

and amplitude-phase form

forced undamped: beating, resonance

forced damped: **x**sp+ **x**tr, practical

resonance

Using conservation of total energy

(=KE+PE) to derive equations of motion,

4.1, 5.1-5.7 linear systems of DEs

first order systems of DEs and tangent vector

fields.

existence-uniqueness thm for first order IVPs

phase portraits for systems of two linear

homogeneous differential equations;

classifications based on eigendata

Vector space theory for linear first order

systems:

superposition, **x**= **x**P+ **x**H

dimension of solution space for **x**H .

conversion of DE IVPs or systems to first

order system IVPs.

Constant coefficient systems and methods:

**x’**(t)=A**x**

**x’**(t)=A**x**+**f**(t)

**x’’**(t)=A**x** (from conservative systems)

**x’’**(t)=A**x**+**f**(t)

Fundamental matrices

Matrix exponentials

Matrix exponential integrating factor for

inhomogeneous systems of first order linear

DEs

applications: phase portrait interpretation of

unforced oscillation problems; input-output

modeling; forced and unforced mass-spring

systems.

7.1-7.6: Laplace transform

definition, for direct computation

using table for Laplace and inverse Laplace

transforms … applications to linear

differential equations and systems of

differential equations from Chapters 3, 5..

Solving linear DE (and system of DE) IVPs with

Laplace transform. Partial fractions, on-off,

convolutions

9.1-9.4: Fourier series

definition, orthogonality and projection.

Computing Fourier series from def. and

rescaling known series

Applications to forced oscillations

Especially for pendulum and mass-spring

Linearization, esp. for pendulum.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

x’’(t) + 5 x’(t) + 4 x(t) = 0 x’’(t) + 5 x’(t) + 4 x(t) = 3 cos(2t)