Math 2280-001

Monday, April 27

Course review

Final exam: Wednesday May 6, 8:00 a.m. -10:00 a.m. (I will let you work until 10:15). This is the official University time and location - our lecture room LCB 215 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. You will be provided a Laplace Transform table and the formulas for Fourier coefficients. The algebra and math on the exam should all be doable by hand.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters.

Chapters:

 1-2: 10-20% first order DEs

 3: 15-30% linear differential equations and applications

 4.1, 5: 30-50% linear systems of differential equations and applications

 7: 15-25% Laplace transforms and applications

 9.1-9.4: 10-15% Fourier series and applications to forced oscillations

On the next page is a more detailed list of the topics we’ve investigated this semester. They are more inter-related than you may have realized at the time, so let’s discuss the connections. Then we’ll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

 slope fields, Euler approximation

 phase diagrams for autonomous DEs

 equilibrium solutions

 stability

 existence-uniqueness thm for IVPs

 methods:

 separable

 linear

 applications

 populations

 velocity-acceleration models

 input-output models

3 Linear differential equations

 IVP existence and uniqueness

 Linear DEs

 Homogeneous solution space,

 its dimension, and why

 superposition, x(t)= xP+ xH

 linear transformations

 aka superposition

 fundamental theorem for solution

 space to L(y)=f when L is linear

 (We use ector space concepts:

 vector spaces and subspaces

 linear combinations

 linear dependence/independence

 span

 basis and dimension)

 Constant coefficient linear DEs

 xH  via characteristic polynomial

 Euler’s formula, complex roots

 xP  via undetermined coefficients

 solving IVPs

 applications:

 mechanical configurations

 unforced: undamped and damped

 cos and sin addition angle formulas

 and amplitude-phase form

 forced undamped: beating, resonance

 forced damped: **x**sp+ **x**tr, practical

 resonance

 Using conservation of total energy

 (=KE+PE) to derive equations of motion,

4.1, 5.1-5.7 linear systems of DEs

 first order systems of DEs and tangent vector

 fields.

 existence-uniqueness thm for first order IVPs

 phase portraits for systems of two linear

 homogeneous differential equations;

 classifications based on eigendata

 Vector space theory for linear first order

 systems:

 superposition, **x**= **x**P+ **x**H

dimension of solution space for **x**H .

 conversion of DE IVPs or systems to first

 order system IVPs.

 Constant coefficient systems and methods:

 **x’**(t)=A**x**

 **x’**(t)=A**x**+**f**(t)

 **x’’**(t)=A**x** (from conservative systems)

 **x’’**(t)=A**x**+**f**(t)

 Fundamental matrices

 Matrix exponentials

 Matrix exponential integrating factor for

 inhomogeneous systems of first order linear

 DEs

 applications: phase portrait interpretation of

 unforced oscillation problems; input-output

 modeling; forced and unforced mass-spring

 systems.

7.1-7.6: Laplace transform

 definition, for direct computation

 using table for Laplace and inverse Laplace

 transforms … applications to linear

 differential equations and systems of

 differential equations from Chapters 3, 5..

 Solving linear DE (and system of DE) IVPs with

 Laplace transform. Partial fractions, on-off,

 convolutions

9.1-9.4: Fourier series

 definition, orthogonality and projection.

 Computing Fourier series from def. and

 rescaling known series

 Applications to forced oscillations

 Especially for pendulum and mass-spring

 Linearization, esp. for pendulum.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

x’’(t) + 5 x’(t) + 4 x(t) = 0 x’’(t) + 5 x’(t) + 4 x(t) = 3 cos(2t)