Math 2280-1 / 2250-4
Tuesday Jan 25
2.1-2.2 cont'd.

- autonomous DE's
  - equilibrium solt'ns
  - stability / asymptotic stability

pages 3-5 Monday notes, including doomsday / extinction example page 5.

additional examples: find equilibria, discuss stability

- $\frac{dx}{dt} = x^4 - x^2$

- $\frac{dx}{dt} = \cos x$

can you think of an example of 1^{st} order autonomous DE with an equilibrium solt'n which is stable, but not also asymptotically stable?

Now, let's harvest a logistic population! (e.g. fisheries).

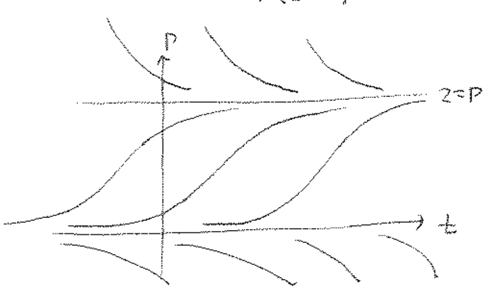
$$\frac{dP}{dt} = \underbrace{aP - bP^2}_{\text{logistic}} \underbrace{- h}_{}$$

constant rate harvesting. see examples 4-6, §2.2

one could also consider a term $-hP$, which would maybe correspond to constant effort harvesting (see §2.2 #23)

e.g. $\frac{dP}{dt} = 2P - P^2$

$= P(2-P)$



vs $\frac{dP}{dt} = 2P - P^2 - h$

consider what happens for different h values

roots of RHS: $P^2 - 2P + h = 0$ has roots

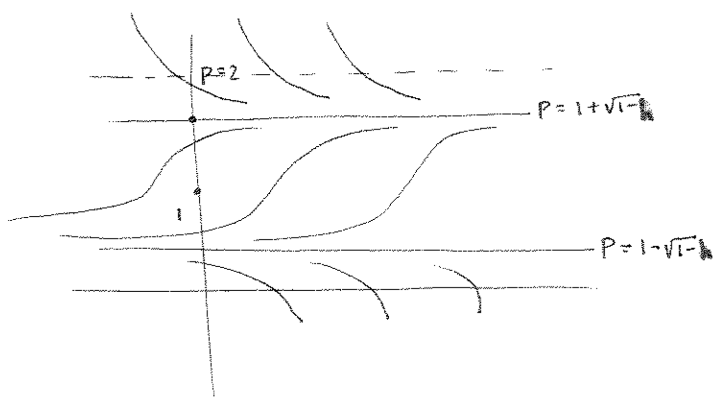
$$P = \frac{2 \pm \sqrt{4-4h}}{2} = 1 \pm \sqrt{1-h}$$

so, for $0 \le h \le 1$,

$$\frac{dP}{dt} = -\left(P - (1+\sqrt{1-h})\right)\left(P - (1-\sqrt{1-h})\right)$$

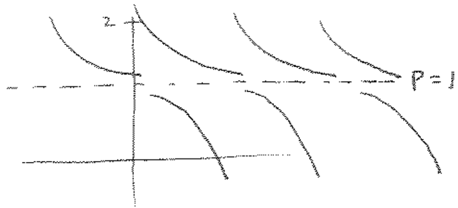
still looks logistic (& fishery won't collapse) if $h \ll 1$

extinction zone

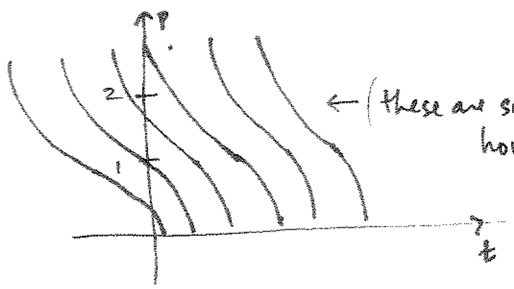


as $h \nearrow 1^-$ the middle zone (where populations increase to the modified carrying capacity) shrinks in width, until at $h = 1$ the two roots coalesce;

$$\frac{dP}{dt} = 2P - P^2 - 1 = -(P^2 - 2P + 1) = -(P-1)^2$$



and for $h > 1$, $\quad 2P - P^2 - h = -(P^2 - 2P + h) = -((P-1)^2 + h - 1) < 0 \quad \forall P$ (but least negative @ $P=1$)

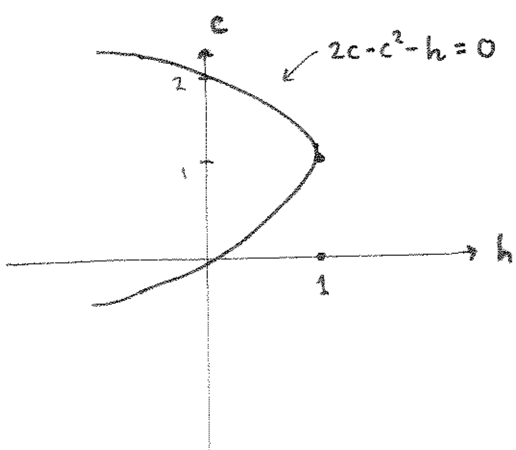


← (these are supposedly horizontal translations of each other)

Extinction $\forall P_0$!!

this model gives a plausible explanation for why fishery after fishery has collapsed around the world – if $h < 1$ but near 1, and if something perturbs the system a little bit (e.g. increased fishing pressure, a big storm, etc.), you could be confronted with "sudden", unexpected fishery collapse.

"bifurcation diagram" of equilibrium sol'ns, in the h-c plane:



$h < 0$ stocking!

$h > 0$ harvesting

(If you fill in the arrows of vertical phase portraits, for varying h, this may make more sense to you)