Math 2280-1
Wed. Jan 13
§1.2-1.3 cont'd
(and a bit of §1.4)

Optional problem session tomorrow for HW due Friday (every Thursday) 8:35-9:35 here!

part of HW for next Friday 1/22

1.3 3, 6, 10, 11, 12, 13
18, 21, 23, 29

6b) also verify $y(x) = x + Ce^{-x}$ solves DE 8
convince yourself the curves you sketch on the slope field are consistent with this formula

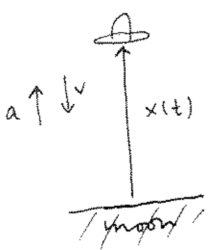
6c) Use dfield to draw the indicated sol'n graphs you drew by hand in 6 (6a), and print out to hand in

29b) get dfield to draw some of these sol'ns & print out to hand in.

§1.2 interesting antidifferentiation examples

Exercise 1 lunar landing example p. 13:

lunar lander descending to moon at speed 450 m/s when (additional) retro rockets are fired, providing deceleration of $2.5\ m/s^2$.
At what height should they be fired for a "perfect" touchdown, i.e. so that velocity $v=0$ exactly when landing happens?

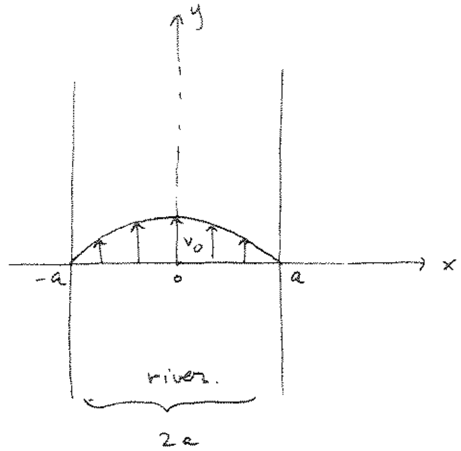


units are important and helpful!

	English	metric
Force	lb	N $(= kg\ m/s^2)$
mass	slug	kg
dist	ft	m
time	s	s
g	$32\ ft/s^2$	$9.8\ m/s^2$

g: accel of grav. on earth

swimmer problem (Examples 3-4 p.15-16). (Exercise 2)



$v_R(x) = v_0(1 - x^2/a^2)$ river velocity

swimmer starts at $(-a, 0)$
and swims due East with const veloc v_s
(aims)

where does swimmer land

$\frac{dx}{dt} = v_s$

$\frac{dy}{dt} = v_R$

2a)
Find the DE:

2b) If river is 1 mile wide and
$v_0 = 9$ mi/h
$v_s = 3$ mi/h

swimmer starts at $(-a, 0)$.
where does swimmer hit opposite shore?

Return to §1.3

well, actually, we'll also use this from §1.4:

"Recall" a separable DE can be written as

$$\frac{dy}{dx} = f(x)\,\phi(y) = \frac{f(x)}{g(y)}$$

math soltn

$$g(y)\frac{dy}{dx} = f(x)$$

$$\frac{d}{dx}\,G(y(x)) = \frac{d}{dx}\,F(x)$$ where $G(y)$ is antider of $g(y)$, $F(x)$ is antideriv of $f(x)$

$$\boxed{G(y) = F(x) + C}$$

expresses y implicitly as a fcn of x

sep of variables algorithm

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$\boxed{G(y) = F(x) + C}$$

You may be able to solve this equation explicitly for $y = y(x)$

Exercise 3a) Use separation of variables to solve $\frac{dy}{dx} = 1 + y^2$

3b) Draw some sol'n graphs onto the slope field

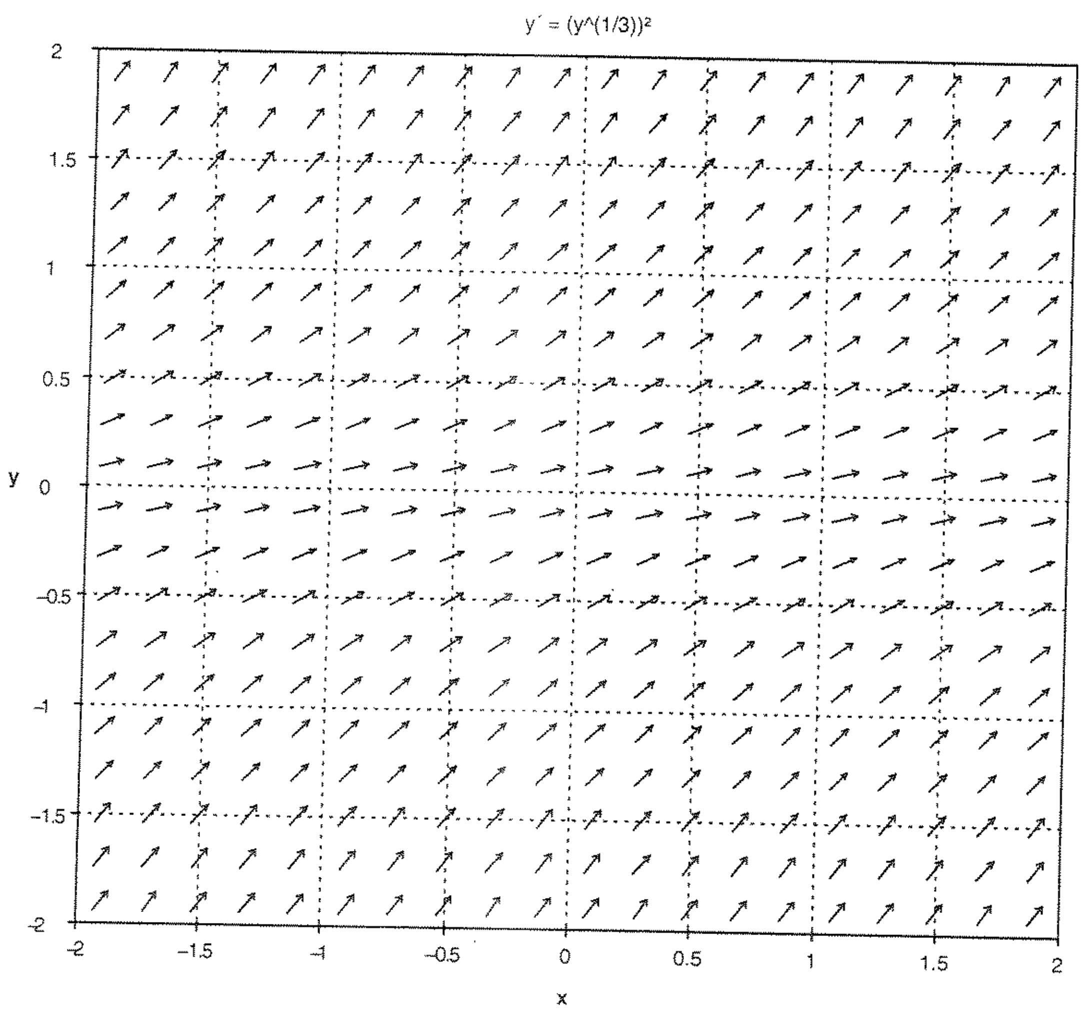
3c) Explain why each IVP has a sol'n, but this sol'n ~~need~~ does not exist for all x.

Exercise 4

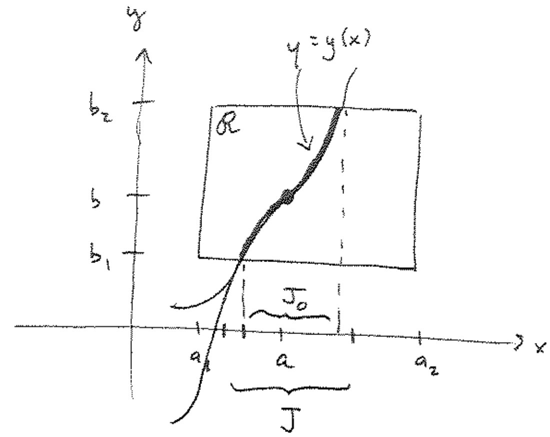
4a) Solve $\begin{cases} \frac{dy}{dx} = y^{2/3} \\ y(0) = 0 \end{cases}$

use separation of variables. Notice this IVP has lots of sol'ns.

4b Sketch some of these onto the slope field.



Usually, existence and uniqueness hold



Theorem 1 $\exists!$: (page 24)

Consider

IVP $\begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$

Let (a,b) be interior to a coord rectangle $\mathcal{R}$
$\left(\begin{array}{c} a_1 \le x \le a_2 \\ b_1 \le y \le b_2 \end{array} \right)$

Let $f(x,y)$ be continuous in $\mathcal{R}$.

Then $\exists$ sol'n to IVP on some interval J containing a in its interior

If $\frac{\partial f}{\partial y}$ exists and is continuous in $\mathcal{R}$ then this solution is unique on any subinterval J_0 s.t. the solution graph lies inside $\mathcal{R}$.

for proof, see appendix

Exercise 5 Discuss how $\exists!$ theorem applies to Exercises 3, 4