

Math 2280-1

Mon. Jan 10

MTWTF 8:35-9:25; LCB 215

Due Jan 14 (this Fri)

1.1 3, ⑥ 15, ①⑥, ①⑨ 27, ③①, ③④, ③⑤, ③⑥

1.2 ⑥ 7 ①③, ①⑧ 19, ②① 25, ②⑨, ③③, ③④, ③⑨

circled problems are mostly graded
uncircled = recommended

• what is a differential equation?

— n^{th} order DE: $F(x, y, y', y'', \dots, y^{(n)}) = 0$

i.e. an equation involving a variable "x" and an (unknown) function $y(x)$ & its first n derivatives

• where do differential eqns come from?

— mathematical models, often of continuous dynamical systems in which the variable "x" is actually time "t".

• goal:

— understand the "solution function(s)" $y(x)$, i.e. the functions for which the differential equation is true.

Examples:

(1) In calculus you have seen

$$\frac{dP}{dt} = kP$$

"x" = t

"y(x)" = P(t)

$k > 0$: population growth rate is proportional to population

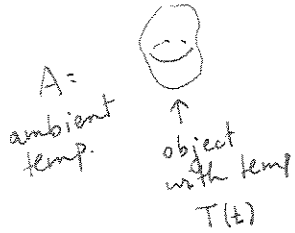
$k < 0$: " decay " " " " "

Solve this DE!

Chain rule backwards

same as differentials (separable DE.)

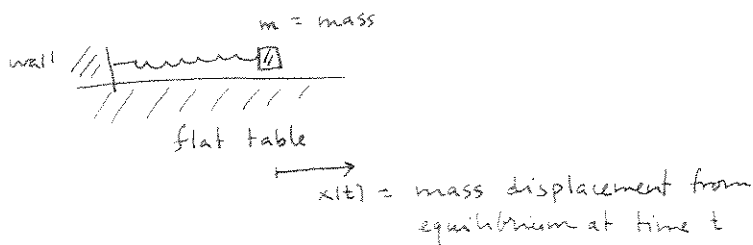
(2) Newton's Law of cooling



$$\frac{dT}{dt} = k(A - T)$$

"rate of change of temp is proportional to difference between temp and ambient temp"

(3) Spring-mass configuration



2nd order DE, from Newton's Law and "linearization"

$$m x''(t) = \underset{\text{mass}}{\text{net force on}} = -kx - cx'$$

mass · acceleration

Example: Consider the no-drag case,

$$* \quad x''(t) + 16x(t) = 0.$$

• Show $x(t) = A \cos 4t + B \sin 4t$ solves this D.E.

• Solve the initial value problem for *,
with initial displacement = 1
initial velocity = 2

↑ If you assume force depends only on position x and velocity x' , $F = F(x, x')$ and $F(0, 0) = 0$ (no net forces at equilibrium)

then this is the linear part of the Taylor series for F , i.e. we ignore higher order terms.

k := Hooke's const
 c := coef of drag.

D

in Math 2270 you probably discussed discrete dynamical systems,
in particular ones of the form

$$\begin{cases} \vec{x}(t+1) = A \vec{x}(t) \\ \vec{x}(0) = \vec{x}_0 \end{cases}$$

e.g. Coyote-roadrunner ?
Glucose-insulin
input-output models
Google

These discrete dynamical systems are related to ODE's we shall
study (starting in chapter 4)...

$$\vec{x}(t+1) = A \vec{x}(t)$$

$$\text{iff } \frac{\vec{x}(t+1) - \vec{x}(t)}{1} = (A - I) \vec{x}(t)$$

$$\text{iff } \frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t} = B \vec{x}(t) \quad (B = A - I, \Delta t = 1)$$

looks like the first order system of DE's

$$\begin{cases} \vec{x}'(t) = B \vec{x}(t) \\ \vec{x}(0) = \vec{x}_0 \end{cases}$$

(2) Newton's Law of cooling

murder mystery... a body is found at 3 A.M. temp = 85°
 by 4 A.M. temp = 80°
 $65^{\circ} = A$

about when did the murder occur?

$$\frac{dT}{dt} = k(A - T) \dots$$