Math 2280-1
Monday Feb. 28

- Finish mass-spring to RLC circuit analogy on Friday notes, for practical "resonance" in circuits.

For more discussion of circuit physics (which ultimately depends on Maxwell's Laws), the sites
http://electronics.howstuffworks.com/oscillator2.htm
wikipedia on electronic circuits

unit correction!

Friday notes wrong !!

Voltage (volts) is actually potential energy (not force), actually the potential energy per unit charge

Kirchoff's Law says that the net changes in PE of a test particle traversing any closed loop in a circuit add up to zero.

Named units derived from SI base units

Name	Symbol	Quantity	Expression in terms of other units	Expression in terms of SI base units
			1/s	s^{-1}
hertz	Hz	frequency	$m \cdot m^{-1}$	dimensionless
radian	rad	angle	$m^2 \cdot m^{-2}$	dimensionless
steradian	sr	solid angle	$kg \cdot m/s^2$	$kg \cdot m \cdot s^{-2}$
newton	N	force, weight	N/m^2	$m^{-1} \cdot kg \cdot s^{-2}$
pascal	Pa	pressure, stress	$N \cdot m = C \cdot V = W \cdot s$	$m^2 \cdot kg \cdot s^{-2}$
joule	J	energy, work, heat	$J/s = V \cdot A$	$m^2 \cdot kg \cdot s^{-3}$
watt	W	power, radiant flux	$s \cdot A$	$s \cdot A$
coulomb	C *	electric charge or quantity of electricity	$W/A = J/C$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
volt	V	voltage, electrical potential difference, electromotive force	C/V	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
farad	F	electric capacitance	V/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$
ohm	Ω	electric resistance, impedance, reactance	$1/\Omega$	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$
siemens	S	electrical conductance	J/A	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
weber	Wb	magnetic flux	$V \cdot s/m^2 = Wb/m^2 = N/(A \cdot m)$	$kg \cdot s^{-2} \cdot A^{-1}$
tesla	T	magnetic field strength, magnetic flux density	$V \cdot s/A = Wb/A$	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
henry	H	inductance	K − 273.15	K − 273.15
Celsius	°C	Celsius temperature	$lx \cdot m^2$	$cd \cdot sr$
lumen	lm	luminous flux	lm/m^2	$m^{-2} \cdot cd \cdot sr$
lux	lx	illuminance	1/s	s^{-1}
becquerel	Bq	radioactivity (decays per unit time)	J/kg	$m^2 \cdot s^{-2}$
gray	Gy	absorbed dose (of ionizing radiation)	J/kg	$m^2 \cdot s^{-2}$
sievert	Sv	equivalent dose (of ionizing radiation)	mol/s	$s^{-1} \cdot mol$
katal	kat	catalytic activity		

http://en.wikipedia.org/wiki/SI_derived_units

* 1 C ≈ charge in 6.24×10^{18} electrons.

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How Oscillators Work

by Marshall Brain

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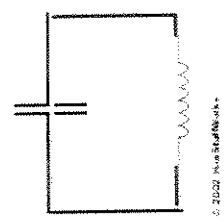
Inside this Article

Oscillator Circuits

Energy needs to move back and forth from one form to another for an oscillator to work. You can make a very simple oscillator by connecting a capacitor and an inductor together. If you've read How Capacitors Work and How Inductors Work, you know that both capacitors and inductors **store energy**. A capacitor stores energy in the form of an electrostatic field, while an inductor uses a magnetic field.

Imagine the following circuit:



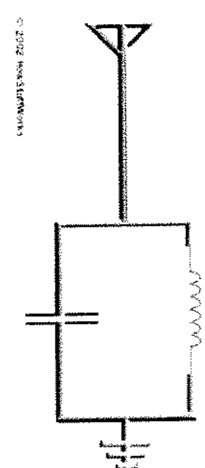
If you charge up the capacitor with a battery and then insert the inductor into the circuit, here's what will happen:

- The capacitor will start to discharge through the inductor. As it does, the inductor will create a magnetic field.
- Once the capacitor discharges, the inductor will try to keep the current in the circuit moving, so it will charge up the other plate of the capacitor.
- Once the inductor's field collapses, the capacitor has been recharged (but with the opposite polarity), so it discharges again through the inductor.

This oscillation will continue until the circuit runs out of energy due to **resistance** in the wire. It will oscillate at a frequency that depends on the size of the inductor and the capacitor.

Resonators

In a simple crystal radio (see How Radio Works for details), a capacitor/inductor oscillator acts as the **tuner** for the radio. It is connected to an antenna and ground like this:



Thousands of **sine waves** from different radio stations hit the antenna. The capacitor and inductor want to resonate at one particular frequency. The sine wave that matches that particular frequency will get **amplified** by the resonator, and all of the other frequencies will be ignored.

In a radio, either the capacitor or the inductor in the resonator is **adjustable**. When you turn the tuner knob on the radio, you are adjusting, for example, a variable capacitor. Varying the capacitor changes the resonant frequency of the resonator and therefore changes the frequency of the sine wave that the resonator amplifies. This is how you "tune in" different stations on the radio!

How Inductors Work

by Marshall Brain

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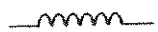
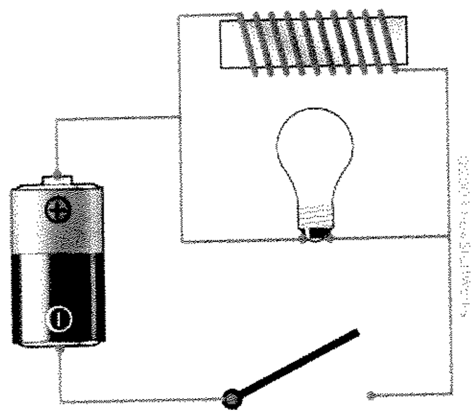
Inductor Basics

In a circuit diagram, an inductor is shown like this:



To understand how an inductor can work in a circuit, this figure is helpful:



What you see here is a battery, a light bulb, a coil of wire around a piece of iron (yellow) and a switch. The coil of wire is an **inductor**. If you have read How Electromagnets Work, you might recognize that the inductor is an electromagnet.

If you were to take the inductor out of this circuit, what you would have is a normal flashlight. You close the switch and the bulb lights up. With the inductor in the circuit as shown, the behavior is completely different.

The light bulb is a **resistor** (the resistance creates heat to make the filament in the bulb glow -- see How Light Bulbs Work for details). The wire in the coil has much lower resistance (it's just wire), so what you would expect when you turn on the switch is for the bulb to glow very dimly. Most of the current should follow the low-resistance path through the loop. What happens instead is that when you close the switch, the bulb burns brightly and then gets dimmer. When you open the switch, the bulb burns very brightly and then quickly goes out.

The reason for this strange behavior is the inductor. When current first starts flowing in the coil, the coil wants to build up a **magnetic field**. While the field is building, the coil inhibits the flow of current. Once the field is built, current can flow normally through the wire. When the switch gets opened, the magnetic field around the coil keeps current flowing in the coil until the field collapses. This current keeps the bulb lit for a period of time even though the switch is open. In other words, an inductor can **store energy** in its magnetic field, and an inductor tends to resist any change in the amount of current flowing through it.

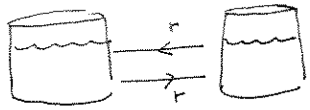
Think About Water...

One way to visualize the action of an inductor is to imagine a narrow channel with water flowing through it, and a heavy water wheel that has its paddles dipping into the channel. Imagine that the water in the channel is not flowing initially.

Now you try to start the water flowing. The paddle wheel will tend to prevent the water from flowing until it has come up to speed with the water. If you then try to stop the flow of water in the channel, the spinning water wheel will try to keep the water moving until its speed of rotation slows back down to the speed of the water. An inductor is doing the same thing with the flow of electrons in a wire -- **an inductor resists a change in the flow of electrons**.

Then begin chapter 4 - systems of differential equations
with some examples.

example 1 tank systems 5.2



$$\frac{dx_1}{dt} = -10\,\frac{x_1}{100} + 10\,\frac{x_2}{50}$$

$$\frac{dx_2}{dt} = 10\,\frac{x_1}{100} - 10\,\frac{x_2}{50}$$

in matrix form, and IVP:

$$\begin{cases} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -.1 & .2 \\ .1 & -.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \underset{=}{\text{e.g.}} \begin{bmatrix} 15 \\ 15 \end{bmatrix} \end{cases}$$

This is an example of a 1^{st} order, constant coefficient, homogeneous linear system of DE's!

$$\text{IVP} \begin{cases} \vec{x}'(t) = A\vec{x} \\ \vec{x}(t_0) = \vec{x}_0 \end{cases}$$

- these IVP's have unique sols (we'll discuss this)
- Sol'n space will be n-dim'l for $A_{n\times n}$ (T: soltn space $\to \vec{x}(t_0)$ is isomorphism)
- look for basis of soltns of the form
  $\vec{x}(t) = e^{\lambda t}\vec{v}$ (analogy to scalar 1^{st} order linear homog DE)
  $\Rightarrow \vec{x}'(t) = \lambda e^{\lambda t}\vec{v}$ ($\vec{v}$ is const)
  if also $\vec{x}'(t) = A\vec{x}$
  deduce $A e^{\lambda t}\vec{v} = \lambda e^{\lambda t}\vec{v}$
  $\quad\;\; \overset{\shortparallel}{\cancel{e^{\lambda t}}A\vec{v}}$ holds iff $A\vec{v} = \lambda\vec{v}$
  ($\vec{v}$ is an eigenvector for A with eigenvalue λ!)

(so if A is diagonalizable we'll get a basis of soltns $\{e^{\lambda_1 t}\vec{v}_1, \ldots e^{\lambda_n t}\vec{v}_n\}$!)

do you already dare to solve the IVP on page 1?

ans: $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \end{bmatrix} - 5e^{-.3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ 3

makes sense!!

Example: restudy a 2^{nd} order (mass-spring) DE
by converting it into a 1^{st} order system of 2 DE's.

2^{nd} order linear homog. DE

$$x'' + .2x' + 1.01x = 0$$

$$x = e^{rt}$$

$$L(x) = e^{rt}\left(\underbrace{r^2 + .2r + 1.01}_{(r+.1)^2 + 1}\right)$$

$$(r+.1+i)(r+.1-i) = 0\ ; \quad r = -.1 \pm i$$

$$x_H(t) = e^{-.1t}\left(A\cos t + B\sin t\right)$$

$$= e^{-.1t}\left(C\cos(t-\alpha)\right)$$

1'st order system of 2 DE's

$$y := x'$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} y \\ -1.01x - .2y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1.01 & -.2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

we'll use the work on the left to solve this system, although it is fun to start the eigenvalue-eigenvector method:

$$\begin{vmatrix} 0-\lambda & 1 \\ -1.01 & -.2-\lambda \end{vmatrix} = \lambda^2 + .2\lambda + 1.01$$

DOES THIS LOOK FAMILIAR ???

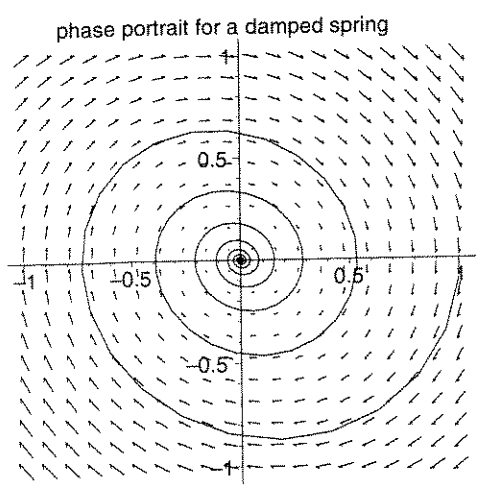
but, rather than dealing with complex eigenvectors today, steal the solution from left column!

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_H \\ x_H' \end{bmatrix}$$

$$= \underbrace{Ce^{-.1t}}_{\text{spirals inward to } \vec{0} \text{ as } t\to\infty.} \underbrace{\begin{bmatrix} \cos(t-\alpha) \\ -.1\cos(t-\alpha) - \sin(t-\alpha) \end{bmatrix}}_{\text{lives on the ellipse}}$$

lives on the ellipse
$$x^2 + (y-.1x)^2 = 1$$
i.e.
$$1.01x^2 - .2xy + y^2 = 1$$

```
with(plots):
arning, the name changecoords has been redefined
pict1:=fieldplot([y,-1.01*x-.2*y],x=-1..1,y=-1..1):
soltn:=plot([exp(-.1*t)*cos(t),exp(-.1*t)*(-.1*cos(t)-sin(t)),t=0.
.400],color=black):
display({pict1,soltn},title='phase portrait for a damped spring');
```



"phase-space picture" for damped mass spring
x = position
(chapter 6)

Class HW problems for Fri March 4

I) Show that the n^{th} order constant coefficient homogeneous DE

$$(1)\quad y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1 y'(t) + a_0 y(t) = 0$$

is related to the 1^{st} order homogeneous system of DE's

$$(2)\quad \begin{bmatrix} x_1(t) \\ x_2 \\ \vdots \\ x_n(t) \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & \cdots & \\ 0 & 0 & 1 & 0 & \cdots \\ & & & & \\ 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

in the sense that:

Ia) If $y(t)$ solves (1), then $\vec{x}(t) := \begin{bmatrix} y(t) \\ y'(t) \\ \vdots \\ y^{(n-1)}(t) \end{bmatrix}$ solves (2)

Ib) (conversely), if $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ solves (2), then $y(t) := x_1(t)$ solves (1)

II) Show that the (eigenvalue) characteristic polynomial $\det(A - rI)$ from the matrix A in (2), is related to the (DE) characteristic polynomial $p(r)$ in (1) by

$$(-1)^n \det(A - rI) = p(r) = r^n + a_{n-1}r^{n-1} + \dots + a_1 r + a_0.$$