Math 2280-1
Friday Feb. 25

Applications day. ü

HW for Friday March 4
3.7 2, 3, 4, 7
4.1 1, 8, 11, 15, 16, 21, 24, 26
2 class exercises TBA
4.3 9 ← work with a classmate if you wish. (Runge Kutta).

1) beating : recall soln to $\begin{cases} x''(t) + \omega_0^2 x = \frac{F_0}{m}\cos\omega t & \omega \neq \omega_0 \\ x(0) = 0 \\ x'(0) = 0 \end{cases}$

is

$$x(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)} \underbrace{(\cos\omega t - \cos\omega_0 t)}_{= -2\sin\left(\frac{\omega-\omega_0}{2} t\right)\sin\left(\frac{\omega+\omega_0}{2} t\right)}$$

$$(\cos(at+bt) - \cos(at-bt) = -2\sin at \sin bt)$$

this is just trigonometry so also applies e.g. to superposition of sound waves

<u>tuning forks</u>
1st: middle C frequency 256 herz (that's not standard; wiki says 261.6 hz.)
2nd: adjustable

2) Continuing discussion of "practical resonance" in slightly damped harmonic oscillators:

derive the formulas we used for $x_{sp}(t)$, when

$$m x'' + c x' + k x = F_0 \cos\omega t \qquad (c > 0).$$

on page 4 Wed.

$$x_{sp}(t) = C\cos(\omega t - \alpha)$$

$$C = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \qquad \alpha = \arccos\left(\frac{k - m^2\omega^2}{\sqrt{(k - m^2\omega^2)^2 + c^2\omega^2}}\right)$$

↑ this is recorded incorrectly in Wed notes

2a

Practical resonance visualization

Math 2280–1
February 25, 2011

```
> restart :
```

$$> C := evalf\left(\frac{F_0}{\mathrm{sqrt}\left(\left(k-m\cdot\omega^2\right)^2+c^2\cdot\omega^2\right)}\right) : \#amplitude$$

$$\text{alpha} := evalf\left(\arccos\left(\frac{\left(k-m\cdot\omega^2\right)}{\mathrm{sqrt}\left(\left(k-m\cdot\omega^2\right)^2+c^2\cdot\omega^2\right)}\right)\right) : \#phase$$

$$\text{delta} := \frac{\text{alpha}}{\text{omega}} : \#\ time\ delay$$

\>

We'll use Maple to look at steady periodic solutions compared to the forcing function for the differential equation

$$\frac{d^2x}{dt^2}+c\cdot\left(\frac{dx}{dt}\right)+26\cdot x(t)=\cos(\omega\cdot t)$$

as we vary the damping coefficient c. We'll also look for potential approximate resonance, like you did in one of your homework problems and we briefly talked about on Wednesday.

```
> with(plots) :

> F0 := 1 :
  k := 26 :
  m := 1 :

>
> omega := 5 :
  c := 0.1 :  #close to natural frequency, and small c
  C;  #should be relatively large
  alpha;
  delta;  #should be close to zero
```

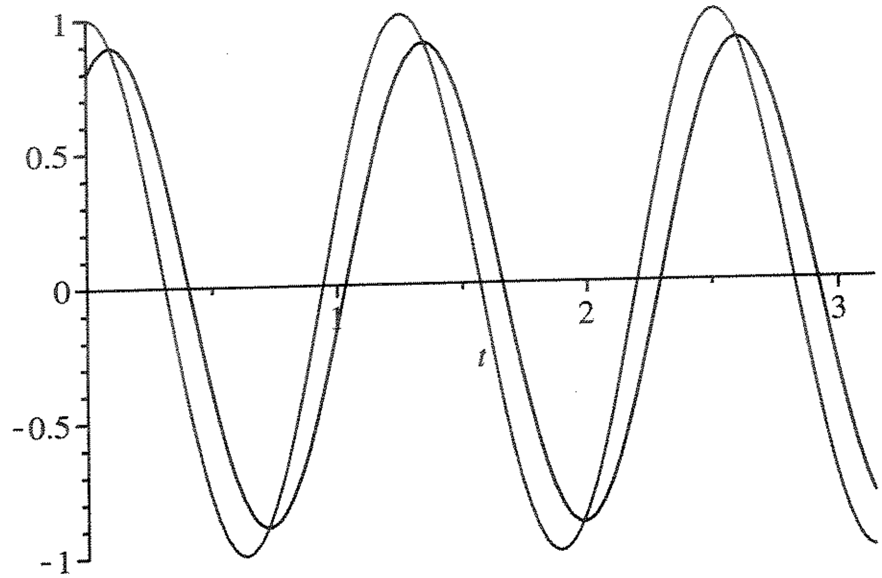
0.8944271908

0.4636476094

0.09272952188 **(1)**

```
> plot1 := plot(cos(omega·t), t = 0 ..Pi, color = blue) :
  plot2 := plot(C·cos(omega·(t − delta)), t = 0 ..Pi, color = black) :
  display({plot1, plot2}, title = 'small damping substantial response picture');
```

small damping substantial response picture

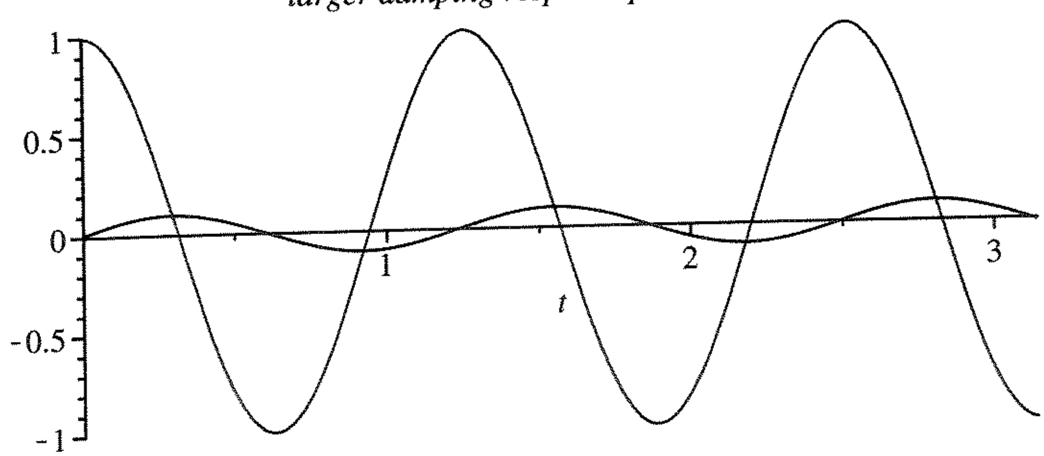


$> \omega := 5:$
$c := 2.:$ *#close to natural frequency, and larger c*
$C;$ *#should be relatively small*
$\alpha;$
$\delta;$ *#more delay*

$$0.09950371903$$
$$1.471127674$$
$$0.2942255348 \qquad (2)$$

$> plot1 := plot(\cos(\omega \cdot t), t = 0..\pi, color = blue):$
$plot2 := plot(C \cdot \cos(\omega \cdot (t - \delta)), t = 0..\pi, color = black):$
display({*plot1*, *plot2*}, *title* = '*larger damping response picture*');

larger damping response picture



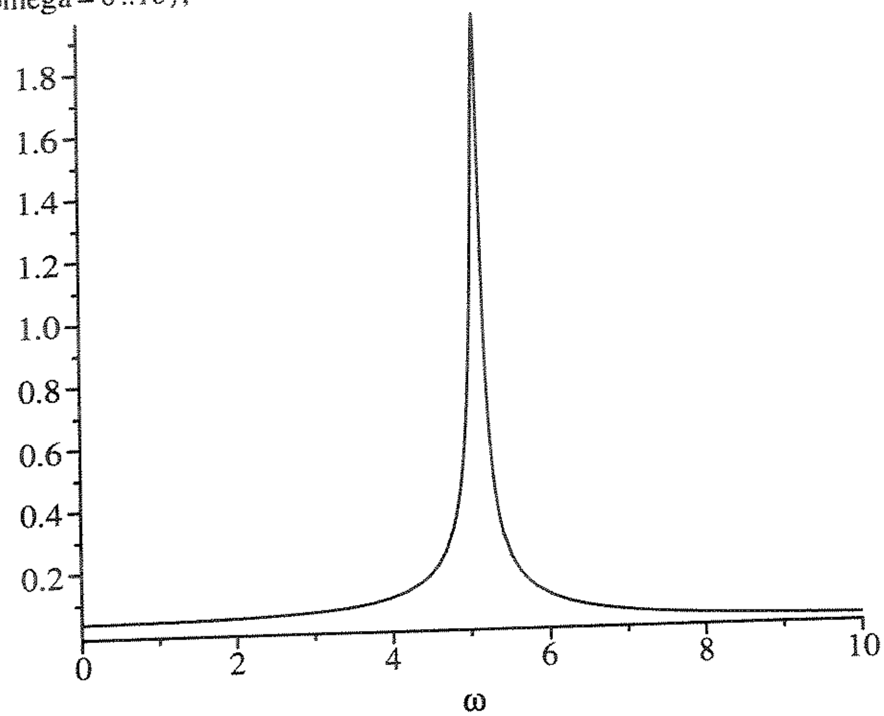
>

practical resonance pictures:

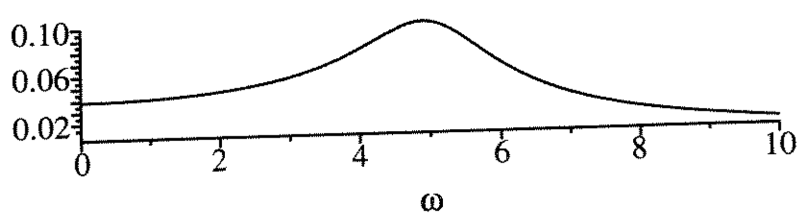
> *unassign*('omega') :

2c

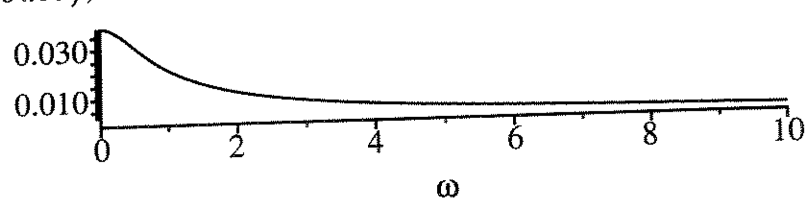
> $c := 0.1$: *#should get peak around omega=sqrt(26):*
plot(C, omega = 0 ..10);



> $c := 2.$: *#book still calls this practical resonance*
plot(C, $\omega = 0 ..10$);

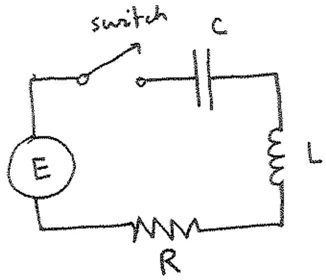


> $c := 40$:
plot(C, $\omega = 0 ..10$);



>

RLC circuits. EP 3.7



voltage E <u>volts</u> (electromotive <u>force</u>)

Circuit elt	Voltage drop	units
Inductor	$L\,\frac{dI}{dt}$	L henries H
Resistor	RI	R ohms Ω
Capacitor	$\frac{1}{C}Q$	C farads F

$Q(t)$ = charge in ~~coulo~~ <u>coulombs</u> (resides on capacitor)

$I(t) = Q'(t)$ = current (<u>amperes</u>) moving around circuit

<u>Ohms Law</u> (like Newton's)

the sum of the voltage drops around a circuit equals the applied voltage E

$$\boxed{\begin{aligned} &LQ'' + RQ' + \tfrac{1}{C}Q = E(t) \\ D_t:\quad &LI'' + RI' + \tfrac{1}{C}I = E'(t) \end{aligned}}$$

mathematically identical to

$$mx'' + cx' + kx = F(t)$$

$$\begin{matrix} \updownarrow & \updownarrow & \updownarrow \\ L & R & 1/C \end{matrix}$$

<u>Exercise 1</u> Set up an IVP for a circuit in which

$R = 16\,\Omega$
$L = 2\,H$
$C = .02\,F$
$E(t) = 100\,V$
$I(0) = 0$
$Q(0) = 5$

Set up one for $Q(t)$ <u>and</u> one for $I(t)$

Example 2 (old) radios — resonance can be good if you're an electrical engineer.

Recall,
Our hard work for damped forced oscillator

$$mx'' + cx' + kx = F_0 \cos\omega t$$

$$x_p = x_{sp}(t) = A\cos\omega t + B\sin\omega t$$

Monday (yesterday) page 2

$$= C\cos(\omega t - \alpha)$$

$$A = F_0 \left(\frac{k-m\omega^2}{(k-m\omega^2)^2 + c^2\omega^2} \right)$$

$$B = F_0 \frac{c\omega}{(k-m\omega^2)^2 + c^2\omega^2}$$

so $$C = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}$$

$0 < \alpha < \pi$

got practical resonance with $\omega \approx \omega_0$, $c \approx 0$.

translate!

note.

$$LQ'' + RQ' + \frac{1}{C}Q = E_0 \sin\omega t$$

D_t: $$LI'' + RI' + \frac{1}{C}I = E_0\omega \cos\omega t$$

steal left column.

$$I_{sp}(t) = I_0 \cos(\omega t - \alpha)$$

$$\left(I_0 = \frac{E_0\omega}{\sqrt{(\frac{1}{C} - L\omega^2)^2 + R^2\omega^2}} \right)$$

$$I_0 = \frac{E_0}{\sqrt{(\frac{1}{\omega C} - L\omega)^2 + R^2}}$$

max response (for fixed E_0, R, ω)
when $\frac{1}{\omega C} = L\omega$

$$\boxed{C = \frac{1}{L\omega^2}}, \quad I_0 = \frac{E_0}{R}$$

old (crystal) radios used the knob to adjust the capacitance to amplify the radio wave!

See p. 230-231 for more