Math 2280-1
Wed Feb. 16.

Exam Friday! ~ tomorrow's problem session 8:35-9:35 to go over 2010 exam

- Begin §3.6: Forced oscillations. (good practice for §3.5, which is on exam).
- Go over review sheet for exam (last page of Tuesday's notes).

Forced oscillations:

$$m x''(t) + c x'(t) + k x(t) = F_0 \cos\omega t$$

$\uparrow$ periodic sinusoidal forcing.

first focus: $c = 0$; no damping

$$x'' + \frac{k}{m} x = \frac{F_0}{m}\cos\omega t$$

$$\text{IVP} \quad \begin{cases} x'' + \omega_0^2 x = \frac{F_0}{m}\cos\omega t \\ x(0) = x_0 \\ x'(0) = v_0 \end{cases} \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

Case 1 $\quad \omega \neq \omega_0$

Use method of undetermined coeff's to show

$$\boxed{x(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)}(\cos\omega t - \cos\omega_0 t) + x_0\cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t}$$

Case 2 Use method of undetermined coeffs to show that for $\omega = \omega_0$ soln to IVP is

$$\boxed{x(t) = \frac{F_0}{2m\omega_0} t \sin\omega_0 t + x_0 \cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t}$$

resonance!

Remark: If $\omega \neq \omega_0$ but $\omega \approx \omega_0$, the phenomenon of "beating" occurs (familiar superposition of functions for musicians)

write $\bar\omega = \frac{\omega+\omega_0}{2}$

$\delta = \frac{\omega-\omega_0}{2}$

then page 1 difference

$$\cos((\bar\omega+\delta)t) - \cos((\bar\omega-\delta)t)$$

$$= \cancel{\cos\bar\omega t \cos\delta t} - \sin\bar\omega t \sin\delta t \;\; \cancel{- \cos\bar\omega t \cos\delta t} - \sin\bar\omega t \sin\delta t$$

$$= -2 \sin\delta t \sin\bar\omega t$$

so page 1 soln

$$x(t) = \frac{F_0}{m(-2\delta)(2\bar\omega)} (-2) \sin\left(\frac{\omega-\omega_0}{2}t\right)\left(\sin\left(\frac{\omega+\omega_0}{2}t\right)\right) + x_0\cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t$$

$$\boxed{x(t) = \frac{F_0}{2m\bar\omega\left(\frac{\omega-\omega_0}{2}\right)} \sin\left(\left(\frac{\omega-\omega_0}{2}\right)t\right) \sin(\bar\omega t) + x_0\cos\omega_0 t + \frac{v_0}{\omega_0}\sin\omega_0 t}$$

slowly oscillating, period $= \frac{2\pi}{\delta}$.

Example:

$F_0 = 5$ $\quad x_0 = 0$

$m = .1$ $\quad v_0 = 0$

$w_0 = 3 \quad (k = .9)$

$$x''(t) + 9x(t) = 50\cos wt$$

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> plot(10*(cos(2*t)-cos(3*t)),t=0..10,color=black);
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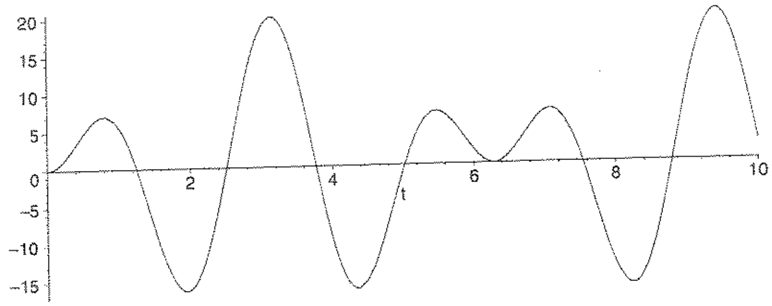
$w=2$: $\dfrac{F_0}{m}\dfrac{1}{w_0^2-w^2}\left[\cos wt - \cos w_0 t\right]$

$= \dfrac{50}{5}\left(\cos 2t - \cos 3t\right)$

(page 1 formula)

(notice this sol'n is periodic, but not sinusoidal)




```
> plot(-100/(9-2.9^2)*sin(-.05*t)*sin(2.95*t),t=0..60,
  numpoints=200,color=black);
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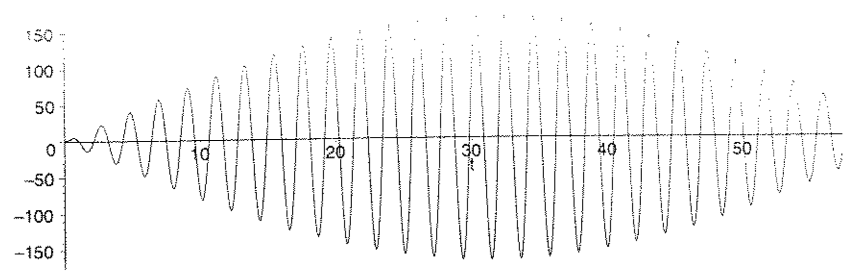
$w = 2.9$ $\quad x(t) = \dfrac{50(-2)}{(9-(2.9)^2)}\sin(-.05t)\sin(2.95t)$

(page 2 formula)

$T = \dfrac{2\pi}{.05} = 40\pi$

notice how large the time-varying "amplitude" is getting!




```
> plot(25/3*t*sin(3*t),t=0..60,numpoints=200,color=black);
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$w = 3$ (page 2 formula) $\quad x(t) = \dfrac{25}{3}t\sin 3t$