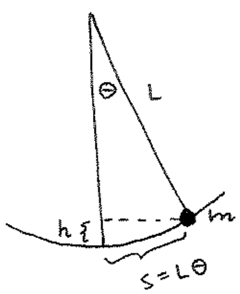
Math 2280-1
Fri. Feb. 11

(§3.4 & 3.5 HW for Wed Feb 16 is posted, and in Wed notes)

Experiment day! (after we finish discussing the harmonic oscillator with damping $c>0$ pages 3-4 Wednesdy notes ~ overdamped, underdamped, critically damped.)

pendulum




conservative system, KE + PE = const.

$$\frac{1}{2}mv^2 + mgh = \text{const}$$

$$s = L\theta \qquad\qquad h = L - L\cos\theta = L(1-\cos\theta)$$

$$v = \frac{ds}{dt} = L\theta'(t)$$

so, $\frac{1}{2}mL^2(\theta')^2 + mgL(1-\cos\theta) \equiv \text{const}$

$D_t$: $mL^2\theta'\theta'' + mgL\sin\theta\,\theta' \equiv 0$

$$\underbrace{mL\theta'}_{\neq 0,\text{ except at isolated times.}}\left(L\theta'' + g\sin\theta\right) \equiv 0$$

~ deduce eqtn of motion is

$$\boxed{\theta'' + \frac{g}{L}\sin\theta = 0}$$

non linear!
but $\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$
$\sin\theta \approx \theta$ for $\theta$ small.
this is an excellent approx.! (alternating series test).

linearize

$$\boxed{\theta'' + \frac{g}{L}\theta = 0}$$

$$\omega_0 = \sqrt{\frac{g}{L}}$$

$$\theta(t) = C\cos(\omega_0 t - \alpha)$$

rediction:

ere is my measurement of L, which we'll check again in class. It assumes the effective length of the ndulum uses the the distance to the ball's center of mass, from the top. (And, where exactly is the p?!)

```
L:=1.528;
g:=9.806;
omega:=sqrt(g/L); #radians per second
f:=evalf(omega/(2*Pi)); #cycles per second
T:=1/f; #seconds per cycle
```

$L := 1.528$

$g := 9.806$

$\omega := 2.533286258$

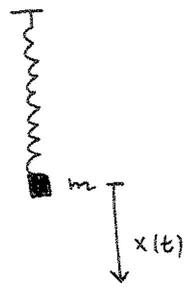
$f := 0.4031850301$

$T := 2.480250816$

Experiment:

Error sources:

## mass – spring




$$m x'' + k x = 0$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

## prediction : (we'll check!)

Mass–spring:
How Hookes constant:

```
> 50.1-6.5; #add 50g and measure displacement
```

$$43.6$$

```
> m:=.1;  #the mass is 100g = 0.1 kg
```

$$m := 0.1$$

My measurement for k, using a 50g mass and measuring the displacement in meters

```
> k*.436=.05*g;
```

$$0.436\, k = 0.49030$$

```
> k:=.05*g/.43;
```

$$k := 1.140232558$$

```
> omega:=sqrt(k/m);#radians per second
  f:=omega/evalf(2*Pi); #cycles per second
  T:=1/f; #seconds per cycle
```

$$\omega := 3.376732974$$

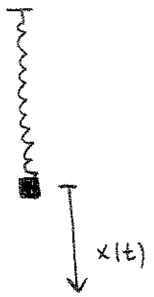
$$f := 0.5374237442$$

$$T := 1.860729100$$

## experiment :

error sources?
(there's a "very" big one!)

Improved spring model (account for spring mass)




normalize KE + PE = 0 for spring hanging at rest. (in gravity!)

then for system in motion

$$KE + PE = \tfrac{1}{2} m (x')^2 + KE_{spring} + \underbrace{\tfrac{1}{2} k x^2}_{\text{PE of system from equilibrium.}}$$

model: the top of the spring isn't moving. the bottom is moving with velocity $x'(t)$.
Assume the velocity of a piece of spring a fraction $\mu$ of the way $(0 \le \mu \le 1)$ from top to bottom is $\mu x'(t)$.

so $\text{speed}^2 = \mu^2 (x')^2$, for that piece. Thus

$$KE = \int_0^1 \tfrac{1}{2} \underbrace{\mu^2 (x'(t))^2}_{v^2} \underbrace{m_s \, d\mu}_{\text{mass}} = \tfrac{1}{6} m_s (x'(t))^2$$

So for $M := m + \tfrac{1}{3} m_s$,

$$KE + PE = \tfrac{1}{2} M (x')^2 + \tfrac{1}{2} k x^2 \equiv \text{const}$$

$D_t$:

$$M x' x'' + k x x' \equiv 0$$

$$x' \big( \underbrace{M x'' + k x}_{=0.} \big) \equiv 0$$

$$\omega_0 = \sqrt{\frac{k}{M}}$$

Does this improve our prediction?

```
> ms:=.011: #spring has mass 11g
  M:=m+ms/3; #"effective mass"
```

$M := 0.1036666667$

```
> omega1:=sqrt(k/M); #new angular freq est.
  f1:=omega1/evalf(2*Pi); #new freq est.
  T1:=1/f1;#new period est
```

$\omega 1 := 3.316478236$

$f1 := 0.5278339048$

$T1 := 1.894535366$

```
>
```