Math 2280-1
Friday April 8

turn HW in, if after 12:00 today & next Friday, to Paula Tooman, JWB 233 (main office), before 5pm

HW for Fri April 15

6.3 8, 9, 10, 14, 15, 16, 17
6.4 12, 13, 14, 15, 16

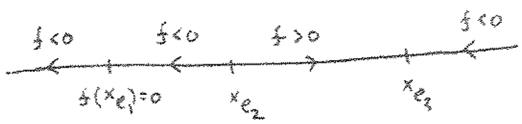
in 6.3.8 and 6.3.10 make a pplane phase portrait for the nonlinear system (3), and explain how your linearization computations are reflected in the non-linear behavior near the corresponding equilibria

7.1 3, 7, 8, 13, 20, 21, 23, 28
7.2 3, 4, 5, 6, 14, 19, 20, 28, 31.

- Finish analysis of predator-prey & competition models, Wed. notes

- Overview of how solution complexity increases as the number of autonomous 1^{st} order DE's in a system increases

<u>$n=1$</u> $\begin{cases} \frac{dx}{dt} = f(x) \\ x(0) = x_0 \end{cases}$



<u>Theorem</u> Let $x(t)$ solve IVP. Then either

(a) $\lim_{t\to\infty} x(t) = \pm\infty$

or (b) $\lim_{t\to\infty} x(t) = x_e$, an equil soln.

(analogous statement for $t \to -\infty$).

and we could prove this!

<u>$n=2$</u> $\begin{cases} \frac{dx}{dt} = f(x,y) \\ \frac{dy}{dt} = g(x,y) \\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \end{cases}$

<u>Theorem</u> Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ solve IVP. Then either

(a) $\lim_{t\to\infty} \left\| \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \right\| = \infty$

(b) $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix}$ an equilibrium soln

<u>or</u>

new possibility! $\rightarrow$ (c) There exists a periodic soln ("cycle") $\begin{bmatrix} \tilde{x}(t) \\ \tilde{y}(t) \end{bmatrix}$)

so that $\lim_{t\to\infty} \left\| \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} - \begin{bmatrix} \tilde{x}(t) \\ \tilde{y}(t) \end{bmatrix} \right\| = 0$.

(this is an interesting theorem to prove, early 1900's, Birkhoff)

<u>$n=3$</u> ($n \geq 3$) : (a), (b), (c), or even "chaotic" behavior, and "strange attractors" §6.5

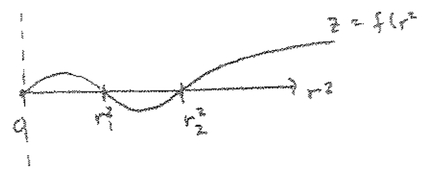
<u>Note</u> : a system $\frac{d\vec{x}}{dt} = \vec{F}(t, \vec{x})$ of k 1^{st} order ~~sy~~ DE's is equivalent to an autonomous system of $(k+1)$ 1^{st} order DE's!

$\begin{cases} \frac{d\vec{x}}{dt} = \vec{F}(T, \vec{x}) \\ \frac{dT}{dt} = 1 \end{cases}$, so e.g. a 2^{nd} order forced oscillation eqtn is like a 1^{st} order autonomous system of 3 DE's. $\sim$ such systems can exhibit chaos

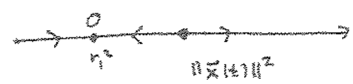
- Examples with possibility (c)

$$* \quad \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -y \\ x \end{bmatrix}}_{\text{this linear system has circular trajectories}} + f(r^2)\begin{bmatrix} x \\ y \end{bmatrix} \qquad r^2 = x^2+y^2$$

$\uparrow$ $Rot_{\pi/2}\begin{bmatrix} x \\ y \end{bmatrix}$ $\qquad$ $\uparrow$ radial field



circular orbits at r_1 & r_2.



for solns to $*$, $\left(\frac{1}{2}(x^2+y^2)\right)' = xx' + yy'$

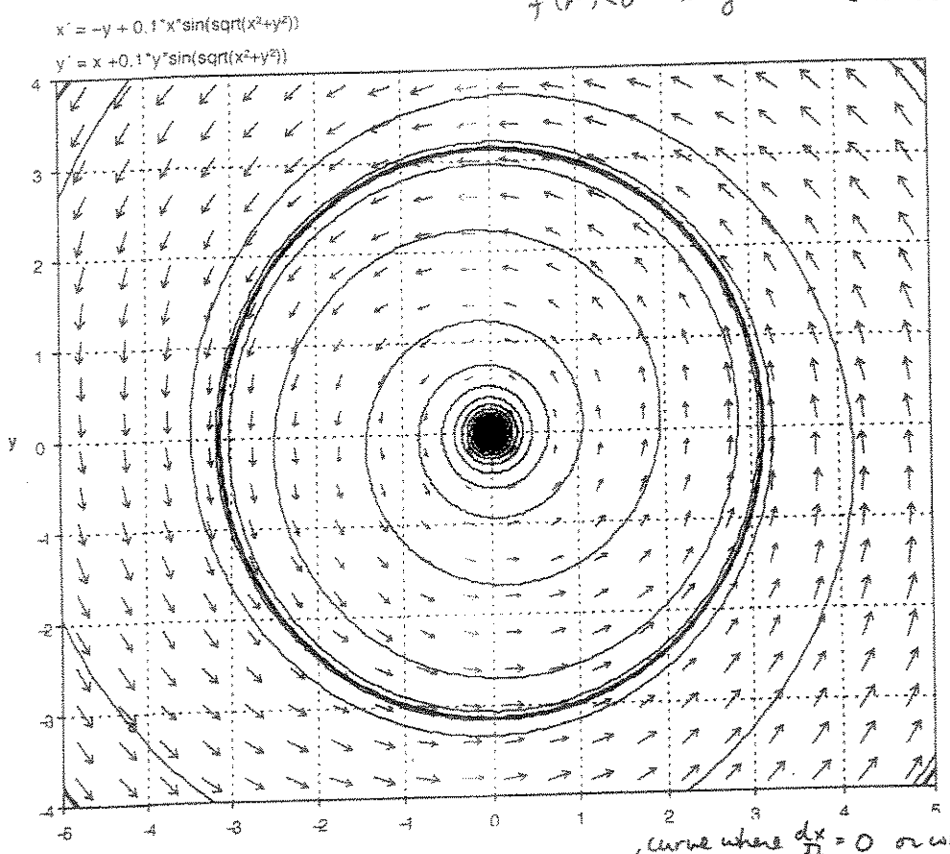
$$= x(\cancel{-y} + f(r^2)x) + y(\cancel{x} + f(r^2)y)$$

$$= f(r^2)(x^2+y^2)$$

so if $f(r^2)=0$, x^2+y^2 is const

$f(r^2)>0$ $\quad x^2+y^2 \nearrow$ (to a zero of f, or to ∞)

$f(r^2)<0$ $\quad x^2+y^2 \searrow$ (to a zero of f, or to 0)



curve where $\frac{dx}{dt}=0$ or where $\frac{dy}{dt}=0$

nullclines intersect at equilibria

$\boxed{\bullet \text{ play with pplane}}$. Discuss "seperatrix", "nullcline", "stable & unstable orbits"

$\uparrow$

solutions converging to saddle pt equilibria, either as $t\to\infty$ or $t\to-\infty$