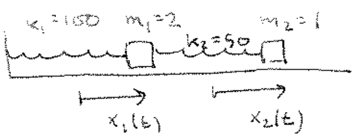
Math 2280-1
Tuesday March 9

§5.2-5.3

- do Glucose-insulin example from this past Friday's notes ~ shows connection between discrete & continuous dynamical systems, & lets us review complex eigenvalues & eigenvectors

- Then begin §5.3: Undamped spring systems

Example 1 p. 321



$$2x_1'' = -100x_1 + 50(x_2 - x_1)$$

$$1\,x_2'' = -50(x_2 - x_1)$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -150 & 50 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$M\vec{x}'' = K\vec{x}$$

in general, with n masses and up to n+1 springs, M & K are n×n matrices

$$\Rightarrow \vec{x}'' = A\vec{x} \quad \text{where } A = M^{-1}K.$$

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$ in this example.

What is the dimension of the solution space? (Careful!)

Look for solutions

$\vec{x}(t) = \cos\omega t\ \vec{v}$ $\overset{?}{\text{or}}$ $\sin\omega t\ \vec{v}$,   based on previous experience and conservation of energy:

if $e^{\lambda t}\vec{v}$ is a solution
$\mathrm{Re}\,\lambda$ must $= 0$ or else solution will either decay or grow exponentially, violating constant total energy!

$$x' = -\omega \sin\omega t\ \vec{v}$$
$$x'' = -\omega^2 \cos\omega t\ \vec{v}$$
$$A\vec{x} = \cos\omega t\ A\vec{v}$$

must have
$$A\vec{v} = -\omega^2\vec{v}$$

$\vec{v}$ an eigenvector of A with eval $\lambda = -\omega^2$.

- hope A has n lin ind evects.
  Each evect gives $\underline{2}$ l.i. solns $\cos\omega t\ \vec{v}$, $\sin\omega t\ \vec{v}$
  $\Rightarrow$ $2n$ lin ind. solns
  $\Rightarrow$ basis.

So, find general soltn to

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Do you get

$$\vec{x}(t) = \underbrace{\left(c_1\cos 5t + c_2\sin 5t\right)}_{C_1\cos(5t-\alpha_1)}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \underbrace{\left(c_3\cos 10t + c_4\sin 10t\right)}_{C_2\cos(10t-\alpha_2)}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Explain what this solution means physically.
If you force this spring system, what angular frequencies for the forcing cosine fcn would induce resonance? (We'll force this system on Wed!)