Math 2280-1
Monday March 1

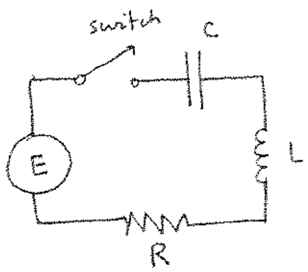
The §4.3 hw problem I assigned on Friday is postponed until next week.

Class exercises A B that I forgot to include in Friday's notes are attached in today's.

- Finish discussion of damped forced oscillations and "practical resonance", p 4-5 Friday's notes

then

- $\boxed{\text{RLC circuits. EP 3.7}}$



voltage E volts (electromotive force)

Circuit elt	Voltage drop	units	
Inductor	$L\frac{dI}{dt}$	L henries	H
Resistor	RI	R ohms	Ω
Capacitor	$\frac{1}{C}Q$	C farads	F

$Q(t)$ = charge in ~~coulo~~ coulombs
(resides on capacitor)

$I(t) = Q'(t)$ = current (amperes)
moving around circuit

Ohms Law (like Newton's)

the sum of the voltage drops around a circuit equals the applied voltage E

$$\boxed{\begin{aligned} &L\,Q'' + R\,Q' + \tfrac{1}{C}Q = E(t) \\ D_t:\quad &L\,I'' + R\,I' + \tfrac{1}{C}I = E'(t) \end{aligned}}$$

mathematically identical to

$$m\,x'' + c\,x' + k\,x = F(t)$$

$m \updownarrow L \qquad c \updownarrow R \qquad k \updownarrow 1/C$

Exercise 1 Set up an IVP for a circuit in which

$R = 16\,\Omega$
$L = 2\,H$
$C = .02\,F$
$E(t) = 100\,V$
$I(0) = 0$
$Q(0) = 5$

Set up one for $Q(t)$
and
one for $I(t)$

Example 2 (old) radios — resonance can be good if you're an electrical engineer.

Recall,
Our hand work for damped forced oscillator

$$mx'' + cx' + kx = F_0 \cos\omega t$$

we just derived

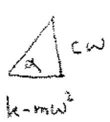
$$x_p = x_{sp}(t) = A\cos\omega t + B\sin\omega t$$
$$= C\cos(\omega t - \alpha)$$

with

$$A = F_0\left(\frac{k-m\omega^2}{(k-m\omega^2)^2 + c^2\omega^2}\right)$$

$$B = F_0 \frac{c\omega}{(k-m\omega^2)^2 + c^2\omega^2}$$

so $C = \dfrac{F_0}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}$



$0 < \alpha < \pi$ since $c\omega > 0$

got practical resonance with $\omega \approx \omega_0$, $c \approx 0$.

translate!

note.

$$LQ'' + RQ' + \frac{1}{C}Q = E_0 \sin\omega t$$

D_t:

$$LI'' + RI' + \frac{1}{C}I = E_0\omega \cos\omega t$$

steal left column.

$$I_{sp}(t) = I_0 \cos(\omega t - \alpha)$$

$$I_0 = \frac{E_0}{\sqrt{\left(\frac{1}{\omega C} - L\omega\right)^2 + R^2}}$$

max response (for fixed E_0, R, ω)

when $\frac{1}{\omega C} = L\omega$

$$\boxed{C = \frac{1}{L\omega^2}}, \quad I_0 = \frac{E_0}{R}$$

old (crystal) radios used the knob to adjust the capacitance to amplify the radio wave!

See p. 230-231 for more

The class homework problems:

(A) Show that the n^{th} order const coeff linear homog DE

$$\text{(1)} \quad y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y'(t) + a_0 y(t) = 0$$

is equivalent to the 1^{st} order homogeneous system

$$\text{(2)} \quad \begin{bmatrix} x_1(t) \\ x_2 \\ \vdots \\ x_n \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots \\ 0 & 0 & \cdots & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \vec{x}'(t) = A\vec{x}$$

in the sense that if $y(t)$ solves (1), then $\begin{bmatrix} y \\ y' \\ y'' \\ \vdots \\ y^{(n-1)} \end{bmatrix}$ solves (2); and

if $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ solves (2), then

$x_1(t)$ solves (1)

(B) Show that the eigenvalue characteristic polynomial $|A - rI|$ from the matrix A in (2), is related to the characteristic poly $p(r)$ is (1), by

$$(-1)^n |A - rI| = p(r) = r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0.$$