Math 2280-1
Wednesday Jan 27

Friday class, meet in LCB 115 to work on your Maple project - I'll have the room open from 8:00-9:35

Recall

<u>autonomous</u> 1^{st} order DE, $\frac{dx}{dt} = f(x)$

<u>equilibrium solutions</u> of DE's:

<u>stability</u>

$x = c$ stable:

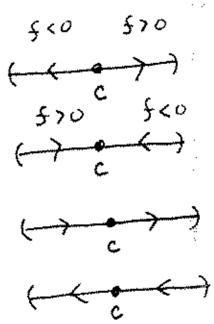
$x = c$ unstable:

$x = c$ asymptotically stable (stronger than stable)
$\exists\ \delta > 0$ s.t. $|x(0) - c| < \delta \Rightarrow \lim_{t \to \infty} x(t) = c$

<u>Theorem</u> Consider the 1^{st} order DE

$$\frac{dx}{dt} = f(x)$$

with $f(x)$ continuously diff'ble in x
Let $x = c$ be an equil sol'tn i.e. $f(c) = 0$.
Then the following <u>local</u> phase portraits imply stability/instability, as indicated.
$t \in I$, some open interval



unstable (implied by $f'(c) > 0$)

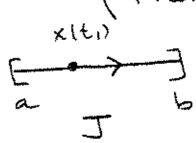
stable (asympt. stable) (implied by $f'(c) < 0$)

unstable (asymp. stable from the left, unstable on the right)

unstable (" " " right, " " left)

This theorem can be proven using the

<u>Lemma</u>: Let $J = [a, b]$ be a closed interval with $f > 0$ on J.
(Then by Calculus extreme value theorem $\exists\ \delta > 0$ s.t. $f(x) \geq \delta\ \forall x \in [a,b]$)



If $\begin{cases} x'(t) = f(x) \\ x(t_1) \in J \end{cases}$

Then $\exists\ t_2 \geq t_1$ with $t_2 \leq t_1 + \frac{b-a}{\delta}$ so that $x(t_2) = b$
(In other words, $x(t)$ increases until it leaves J).
(Analogous lemma for $f < 0$, J, in which case $x(t)$ decreases until it leaves J)

prove the lemma and explain the theorem!

- Then solve the doomsday/extinction problem page 4 Tuesday.

Sinusoidal function essentials

(for first Maple project & later, see §3.4)

These notes explain how to rewrite the linear combo

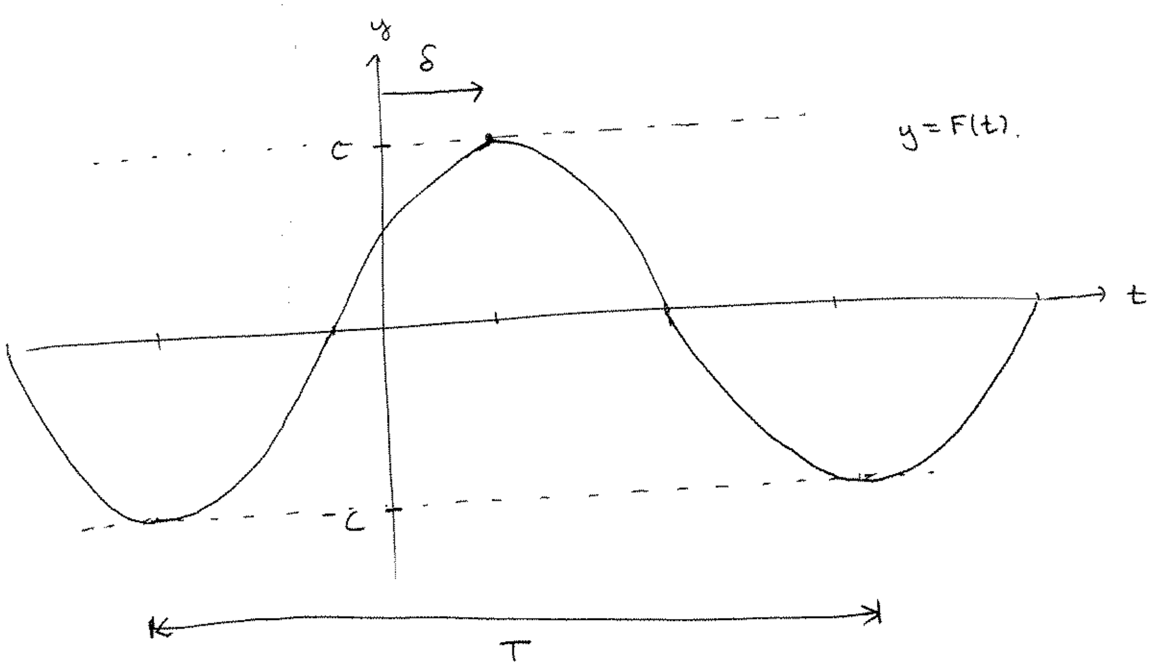
$$F(t) = A\cos\omega t + B\sin\omega t$$

in amplitude phase form

$$F(t) = C\cos(\omega t - \alpha) \qquad C > 0$$
$$= C\cos(\omega(t-\delta)) \qquad \alpha = \omega\delta, \quad \delta = \alpha/\omega$$

C = amplitude
ω = angular frequency radians/time
α = phase angle
δ = time delay ~ graph of F is graph of $C\cos\omega t$ translated δ units to the right.

f = frequency $= \frac{\omega}{2\pi}$ cycles/sec (or other time unit).
T = period $= 1/f = \frac{2\pi}{\omega}$ sec/cycle



Recall $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Thus

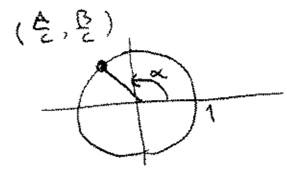
$$C\cos(\omega t - \alpha) = C\cos\omega t\cos\alpha + C\sin\omega t\sin\alpha$$
$$\stackrel{?}{=} A\cos\omega t + B\sin\omega t \qquad \text{as fcns of } t$$

possible iff

$$A = C\cos\alpha$$
$$B = C\sin\alpha$$

iff

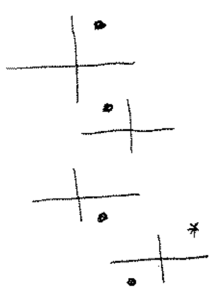
$$\frac{A}{C} = \cos\alpha$$
$$\frac{B}{C} = \sin\alpha$$



there exists such an angle iff

$$\left(\frac{A}{C}\right)^2 + \left(\frac{B}{C}\right)^2 = 1 \qquad \text{i.e.} \qquad \boxed{C = \sqrt{A^2+B^2}}$$

and then use inverse trig to find α



$(A,B) \in 1^{st}$ quadrant $\quad \alpha = \arctan\frac{B}{A} = \arcsin\left(\frac{B}{C}\right) = \arccos\left(\frac{A}{C}\right)$

$(A,B) \in 2^{nd}$ " $\quad \alpha = \arccos\left(\frac{A}{C}\right)$

$(A,B) \in 4^{th}$ " $\quad \alpha = \arctan\left(\frac{B}{A}\right) = \arcsin\left(\frac{B}{C}\right)$

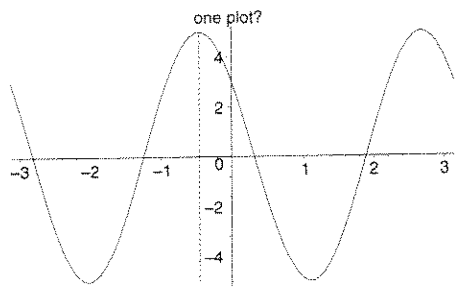
$(A,B) \in 3^{rd}$ " $\quad \alpha = \arctan\left(\frac{B}{A}\right) + \pi$

<u>Example</u> Express $F(t) = 3\cos 2t - 4\sin 2t$ in phase amplitude form

<u>ans</u> $F(t) = 5\cos(2t + .927)$

$\left(= 5\cos(2t + (.927 - 2\pi)\right)$

```
> with(plots):
  plot1:=plot(3*cos(2*t)-4*sin(2*t),t=-Pi..Pi,color=black):
  plot2:=plot(5*cos(2*t+.927),t=-Pi..Pi,color=red):
  plot3:=plot([-.46,t,t=-5..5],linestyle=2,color=black):
  display({plot1,plot2,plot3},title='one plot?');
```




```
> ?plot   #help window to help me figure out to draw
          # the dotted line above, to show graph translation
```