

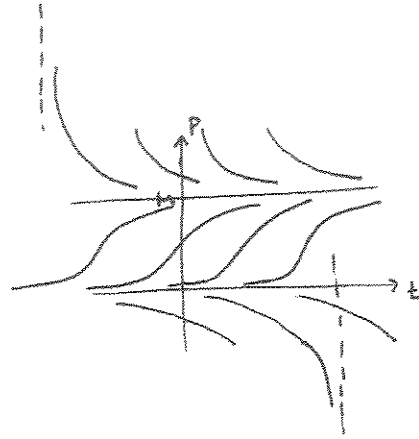
Math 2280-1

Tuesday Jan 26.

- Yesterday we solved the logistic IVP:

$$\text{IVP} \begin{cases} \frac{dP}{dt} = kP(M-P) \\ P(0) = P_0 \end{cases}$$

$$\begin{aligned} \text{soltn: } P(t) &= \frac{M}{1 + \left(\frac{M-P_0}{P_0}\right)e^{-Mkt}} \\ &= \frac{MP_0}{P_0 + (M-P_0)e^{-Mkt}} \end{aligned}$$



if you set denom = 0

$$P_0 = (P_0 - M)e^{-Mkt}$$

$$\frac{P_0}{P_0 - M} = e^{-Mkt}$$

if $P_0 > 0$ then for
 $0 < P_0 < M$ there
 are no t -values
 making eqn true,
 since LHS is negative

if $P_0 > M$ there is a
 negative t -value to
 making eqn true,
 $\lim_{t \rightarrow t_0^+} P(t) = +\infty$

if $P_0 < 0$ there is a
 positive t -value to
 making eqn true,
 $\lim_{t \rightarrow t_0^-} P(t) = -\infty$

- Go over U.S. population modeling from Monday handout.

(2)

Before discussing more population (and also velocity-acceleration) models,
let's talk about the general language that gets used...

§2.2: A general 1st order DE $\frac{dx}{dt} = f(t, x)$

is called autonomous if $\frac{dx}{dt} = f(x)$

($\frac{dx}{dt}$ only depends on x itself, not also on t)

def $x(t)$ is an equilibrium soln to a DE iff $x(t) \equiv C$, a constant

If the DE is autonomous and $x(t) \equiv C$ is an equilibrium soln, then

$$0 = \frac{dx}{dt} = f(x) = f(C) = 0.$$

And if $f(C) = 0$ then $x(t) \equiv C$ is an equilibrium soln.

equilibrium sol's of
 $\frac{dx}{dt} = f(x)$ are exactly the
fns $x(t) \equiv C$
where $f(C) = 0$

example

$$\frac{dx}{dt} = kx(M-x)$$

$x \equiv 0$
 $x \equiv M$ are the equil. sol's

example find the equil. sol's of

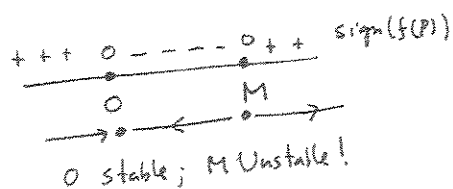
$$\frac{dx}{dt} = x^3 + 2x^2 + x$$

More "real" examples

doomsday extinction:

$$\frac{dP}{dt} = aP^2 - bP \quad a, b > 0.$$

$$= kP(P-M) \quad k=a, -kM=b$$



e.g. if $\beta(t)$ (fertility rate) $\sim P$

(e.g. herd roams in a fixed area and mates randomly upon meeting opposite-sex mate)

then $B(t) = aP^2$;

perhaps $D(t) = -bP$.

If $0 < P_0 < M$ then $\lim_{t \rightarrow \infty} P(t) = 0$ (so in real life, extinction)
 If $P_0 > M$ then $\exists t_1 < \infty$ s.t. $\lim_{t \rightarrow t_1} P(t) = +\infty$ (doomsday!)

example

$$\begin{cases} \frac{dx}{dt} = x(x-1) \\ x(0) = 2 \end{cases}$$

Find the time of doomsday.

(ans $t = \ln 2$!)

