

Math 2280-1

Wednesday Jan 20

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- have you checked page 4 Tuesday, for cylindrical water tank:
If T_μ is the time it takes to drain from height h to height μh in a cylinder,

then $T_0 = \frac{T_\mu}{1-\sqrt{\mu}}$

- do the experiment !!
 - time $T_{1/2}$
 - estimate T_0 from our model.
 - time T_0
 - compute relative error between experiment and theory.
what measurement & model errors could contribute?

§ 1.5 Linear 1st order DE's.

$$(1) \quad \frac{dy}{dx} + P(x)y = Q(x)$$

Notice the left side of this eqn, $L(y) := y' + P(x)y$ is linear:

$$L(y_1 + y_2) = L(y_1) + L(y_2)$$

(y_1, y_2 diffble)

$$L(cy) = cL(y)$$

(c a const, y diffble)

Solution method is to multiply both sides by a non-zero fn ("integrating factor") so that we can antidifferentiate wrt x to deduce $y(x)$:

eqn (1) is equiv. to

$$(2) \quad e^{\int P(x)dx} (y' + P(x)y) = e^{\int P(x)dx} Q(x)$$

where $\int P(x)dx$ is any particular antideriv. of $P(x)$

equiv to

$$(3) \quad \frac{d}{dx} \left[e^{\int P(x)dx} y \right] = e^{\int P(x)dx} Q(x)$$

or (antidifferentiating)

$$(4) \quad e^{\int P(x)dx} y = \int \left[\underbrace{e^{\int P(x)dx} Q(x)} \right] dx \quad ; \text{ divide by } e^{\int P(x)dx} \text{ to get } y(x)$$

Example HW problem #6 § 1.3 you are asked to plug in $y = x + Ce^{-x}$ to show it solves the DE

$$y' = x - y + 1$$

Use the algorithm above to derive the given soltns:

$$y' + 1y = x + 1$$

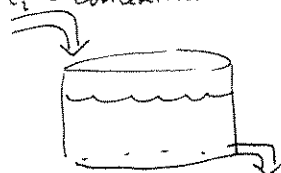
example 2 page 48-49:

$$(x^2+1) \frac{dy}{dx} + 3xy = 6x$$

$$\text{ans } y = 2 + C(x^2+1)^{-3/2}$$

• Example: mixing problems (p. 51) applies also to pharmacology & environment

r_i = rate in ℓ/s
 c_i = concentration in gm/ℓ



r_o = rate out ℓ/s
 c_o = concentration out gm/ℓ

$V(t)$ = volume in tank at time t (ℓ)

$x(t)$ = amount of solute in tank (gm)

$c(t) = \frac{x(t)}{V(t)}$ $\frac{g}{\ell}$ (average) concentration in tank

Assume mixture is uniform so $c(t)$ is spatially constant

$$\frac{dV}{dt} = r_i - r_o \quad \ell/s \quad \rightarrow \quad V(t) = V_0 + \int_0^t r_i(s) - r_o(s) ds \quad \S 1.2$$

$$\begin{aligned} \frac{dx}{dt} &= r_i c_i - r_o c_o && \frac{g}{s} \quad g/s \\ &= r_i c_i - r_o \frac{x(t)}{V(t)} && \text{by "well mixed" model} \end{aligned}$$

$$\boxed{\frac{dx}{dt} + \frac{r_o}{V(t)} x(t) = r_i c_i}$$

$\uparrow$ $\uparrow$
 $P(t)$ $Q(t)$

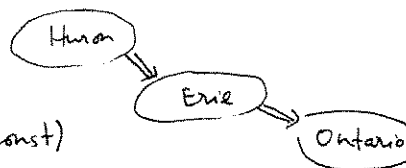
Example 4 p. 52 Lake Erie pollution

$$V(t) = 480 \text{ km}^3$$

$$r = r_i = r_o = 350 \text{ km}^3/\text{year} \quad (\text{so } V \text{ is const})$$

$c_i = c$ = pollutant conc. of lake Huron

$$x_o = (5c)V \quad (\text{read problem}).$$



When will Erie pollution concentration have decreased to twice that of Huron?

(5)

i.e. solve, and analyze

$$\begin{cases} \frac{dx}{dt} = r_i c_i - r_o c_o \\ x(0) = 5cV \end{cases}$$

$$\frac{dx}{dt} = rc - r \frac{x}{V}$$

for fun, notice this DE is also separable.
Can you solve it that way?

$$\frac{dx}{dt} + \frac{r}{V} x = rc$$

$$(e^{\frac{r}{V}t} x)' = rc e^{\frac{r}{V}t}$$

$$e^{\frac{r}{V}t} x = rc \frac{V}{r} e^{\frac{r}{V}t} + C$$

$$x = cV + C e^{-\frac{r}{V}t}$$

$$x(0) = 5cV \Rightarrow C = 4cV$$

$$x(t) = cV + 4cV e^{-\frac{r}{V}t}$$

$$\text{set } x(T) = 2cV = cV + 4cV e^{-\frac{r}{V}T}$$

$$.25 = e^{-\frac{r}{V}T}$$

$$-\frac{r}{V}T = \ln(.25)$$

$$T = -\ln(.25) \frac{V}{r}$$

$$= \ln(4) \frac{480}{350} \text{ years}$$

$$\boxed{\approx 1.90 \text{ years}}$$