

Math 2280-1

Wed. Jan 13

§1.2-1.3 cont'd
(and a bit of §1.4)

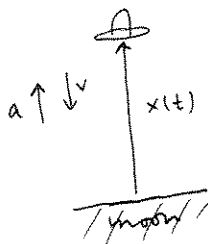
Optional problem
session tomorrow for
HW due Friday
(every Thursday)
8:35-9:35 JTB 120

§1.2 interesting antidifferentiation examples

Exercise 1 lunar landing example p. 13:

lunar lander descending to moon at
speed 450 m/s when (additional)
retro rockets are fired, providing deceleration
of 2.5 m/s^2 .

At what height should they be fired for
a "perfect" touchdown, i.e. so that
velocity $v=0$ exactly when landing happens?



part of HW for next Friday
1/22

1.3 3, (6), 10, 11, 12, 13

(18, 21, 23, 29)

* (6b) also
verify $y(x) = x + Ce^{-x}$
solves DE &
convince yourself
the curves you sketch
on the slope field
are consistent with
this formula

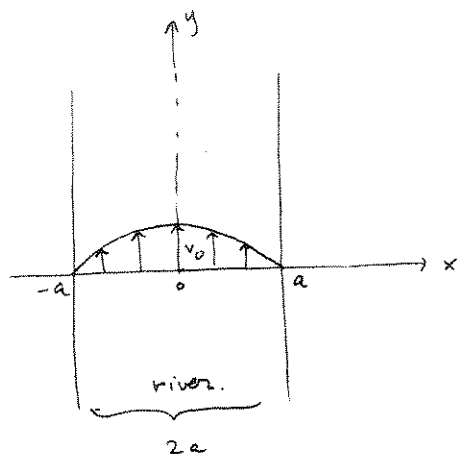
(29b)
get dfield
to draw
some of
these
sol'n's &
print out
to hand
in.

(6c) Use dfield
to draw the
indicated sol'n
graphs you drew
by hand in 6 (6a),
and print out to hand in

units are important
and helpful!

	English	metric
Force	lb	N ($= \text{kg} \cdot \text{m/s}^2$)
mass	slug	kg
dist	ft	m
time	s	s
	32 ft/s ²	9.8 m/s ²
g ↓ accel of grav. on earth		

swimmer problem (Examples 3-4 p. 15-16). (Exercise 2)



$$v_R(x) = v_0 \left(1 - \frac{x^2}{a^2}\right) \quad \text{river velocity}$$

swimmer starts at $(-a, 0)$
and swims due East with const veloc v_s
(aims)

where does swimmer land

$$\frac{dx}{dt} = v_s$$

2a)

Find the DE:

$$\frac{dy}{dt} = v_R$$

2b) If river is 1 mile wide and
 $v_0 = 9$ mi/h
 $v_s = 3$ mi/h

swimmer starts at $(-a, 0)$.
where does swimmer hit opposite shore?

Return to §1.3

well, actually, we'll also use this from §1.4:

"Recall" a separable DE can be written as

$$\frac{dy}{dx} = f(x)g(y) = \frac{f(x)}{g(y)}$$

sep of variables
algorithm

math soln

$$g(y) \frac{dy}{dx} = f(x)$$

$$g(y) dy = f(x) dx$$

$$\int g(y) dy = \int f(x) dx$$

$$\boxed{G(y) = F(x) + C}$$

$$\frac{d}{dx} G(y(x)) = \frac{d}{dx} F(x) \quad \text{where } G(y) \text{ is antideriv of } g(y)$$

$F(x)$ is antideriv of $f(x)$

$$\boxed{G(y) = F(x) + C}$$

expresses y implicitly as a fun of x

You may be able to solve this equation explicitly for $y = y(x)$

Exercise 3a) Use separation of variables

to solve $\frac{dy}{dx} = 1 + y^2$

3b) Draw some sol'n graphs onto the slope field

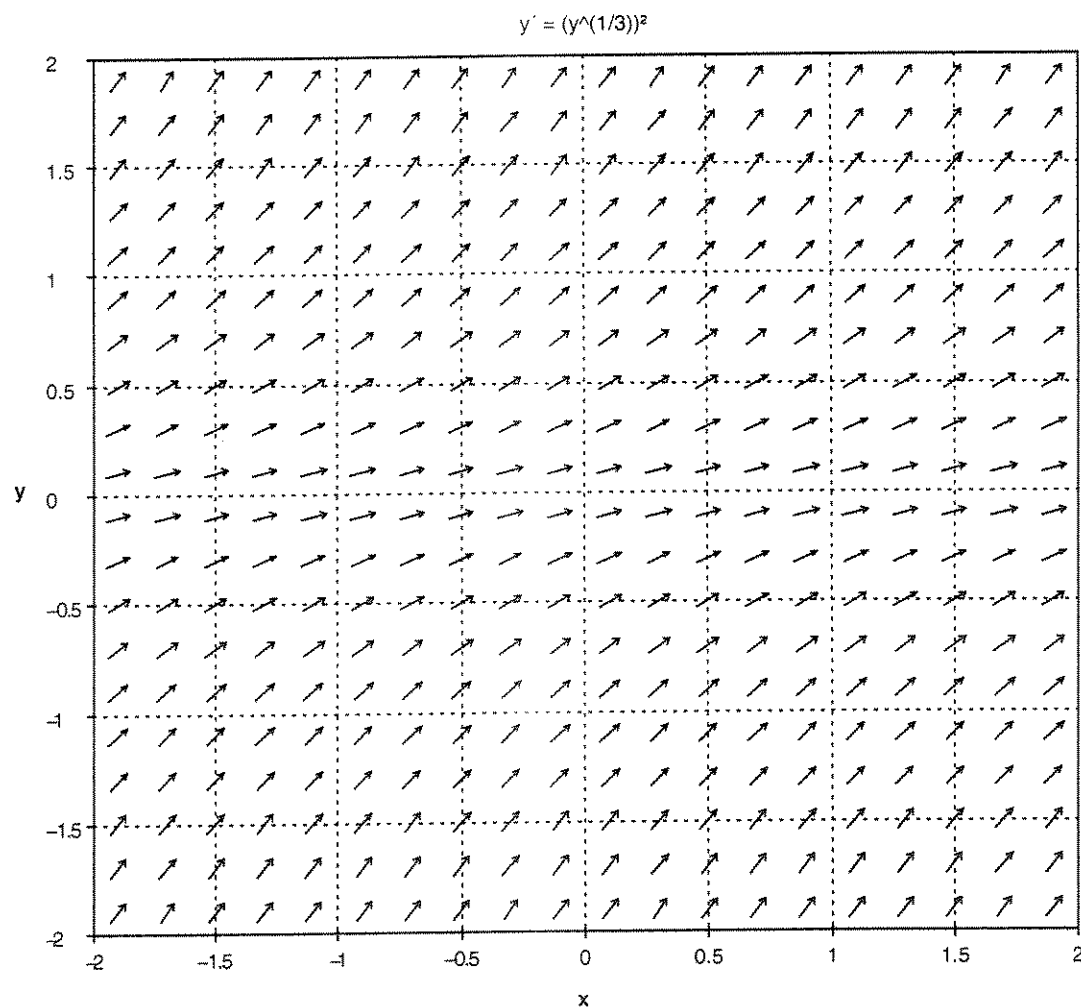
3c) Explain why each IVP has a sol'n, but this sol'n ~~does~~ does not exist for all x .

Exercise 4

4a) Solve $\begin{cases} \frac{dy}{dx} = y^{2/3} \\ y(0) = 0 \end{cases}$

use separation of variables. Notice this IVP has lots of solns.

4b Sketch some of these onto the slope field.



Usually, existence and uniqueness hold

Theorem 1 $\exists!$: (page 23)

Consider

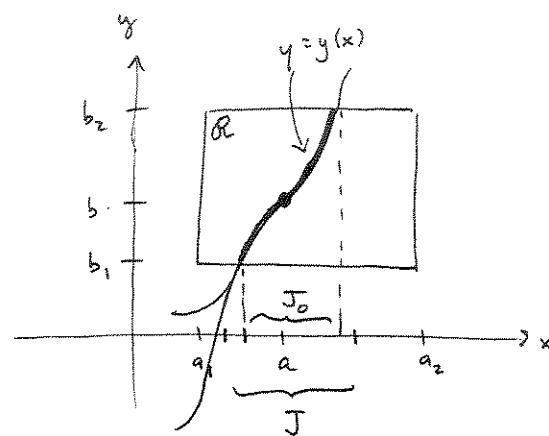
$$\text{IVP} \begin{cases} \frac{dy}{dx} = f(x, y) \\ y(a) = b \end{cases}$$

Let (a, b) be interior to a cond rectangle R
($a_1 \leq x \leq a_2$)
 $b_1 \leq y \leq b_2$

Let $f(x, y)$ be continuous in R .

Then $\exists$ sol'n to IVP on some interval J
containing a in its interior

If $\frac{\partial f}{\partial y}$ exists and is continuous in R then this
solution is unique on any subinterval J_0
s.t. the solution graph lies inside R .



for proof see appendix

Exercise 5 Discuss how $\exists!$ theorem applies to Exercises 3, 4