

Tuesday 12 Jan.

§1.2-1.3

①

- discuss the connection between discrete dynamical systems (2270), and system of differential equation, the continuous analog; page 3 Monday

- solve the murder mystery, p. 4 Monday notes

Then begin §1.2-1.3: A 1st order DE is an equation $F(x, y, \frac{dy}{dx}) = 0$

e.g. $xy + (y')^2 = 0$

can usually use algebra to rewrite as

$$\frac{dy}{dx} = f(x, y)$$

If we also want to specify

$$y(x_0) = y_0$$

Then the DE together with the specified y -value ("initial value") is called an initial value problem:

$$\text{IVP} \begin{cases} \frac{dy}{dx} = f(x, y) \\ y(x_0) = y_0 \end{cases}$$

example 1:

$$\text{IVP} \begin{cases} \frac{dy}{dx} = x - 3 \\ y(1) = 2 \end{cases}$$

Notice: this DE is of the form $y' = f(x)$

(y' only depends on x , not also on $y(x)$)

(such DE's are focus of §1.2)

You can solve this IVP by antidifferentiation:

$$\text{ans: } y = \frac{x^2}{2} - 3x + \frac{9}{2}$$

example 2

$$\text{IVP} \begin{cases} \frac{dy}{dx} = y - x \\ y(0) = 0 \end{cases}$$

} magic (actually §1.5) (not §1.2)

$$y = x + 1 + Ce^x \text{ solves DE}$$

check:

solve IVP:

$$\text{ans: } y = x + 1 - e^x$$

slope fields

for any D.E. of the form

$$(*) \quad \frac{dy}{dx} = f(x, y)$$

geometric interpretation of (*):

If $y(x)$ solves (*), the

slope of its graph $y=y(x)$ at $(x, y(x))$

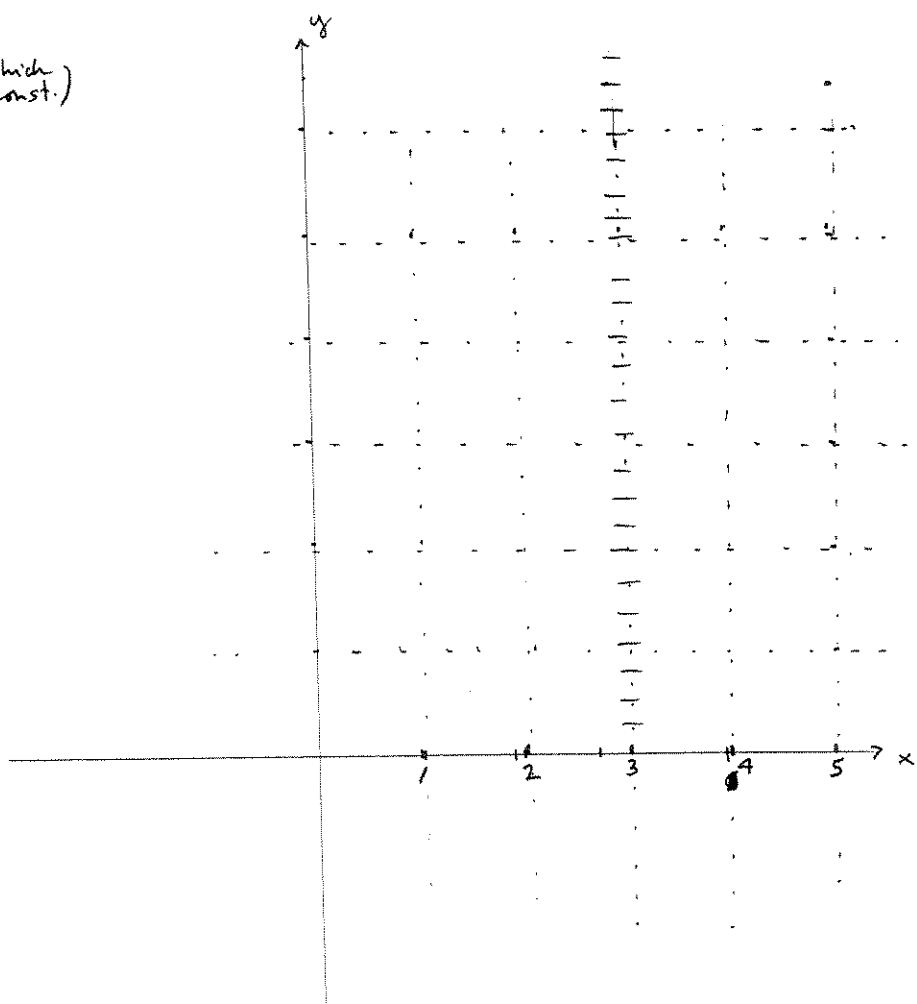
is determined by the point $(x, y(x))$, i.e. slope = $f(x, y)$

So if you draw a picture of the slope field $m=f(x, y)$ in the x - y plane, solution graphs $y=y(x)$ will be tangent to the field.

example 1 : • draw the slope field corresponding to the DE $\frac{dy}{dx} = x-3$; i.e. $m(x, y) = x-3$

• sketch the IVP sol'n from page 1 onto this slope field.

slope value m	equation of corresponding isocline (curve on which slope is const.)
0	$x-3=0$ or $x=3$
1	$x=4$
2	$x=5$
-1	$x-3=-1$ or $x=2$
-2	$x=1$



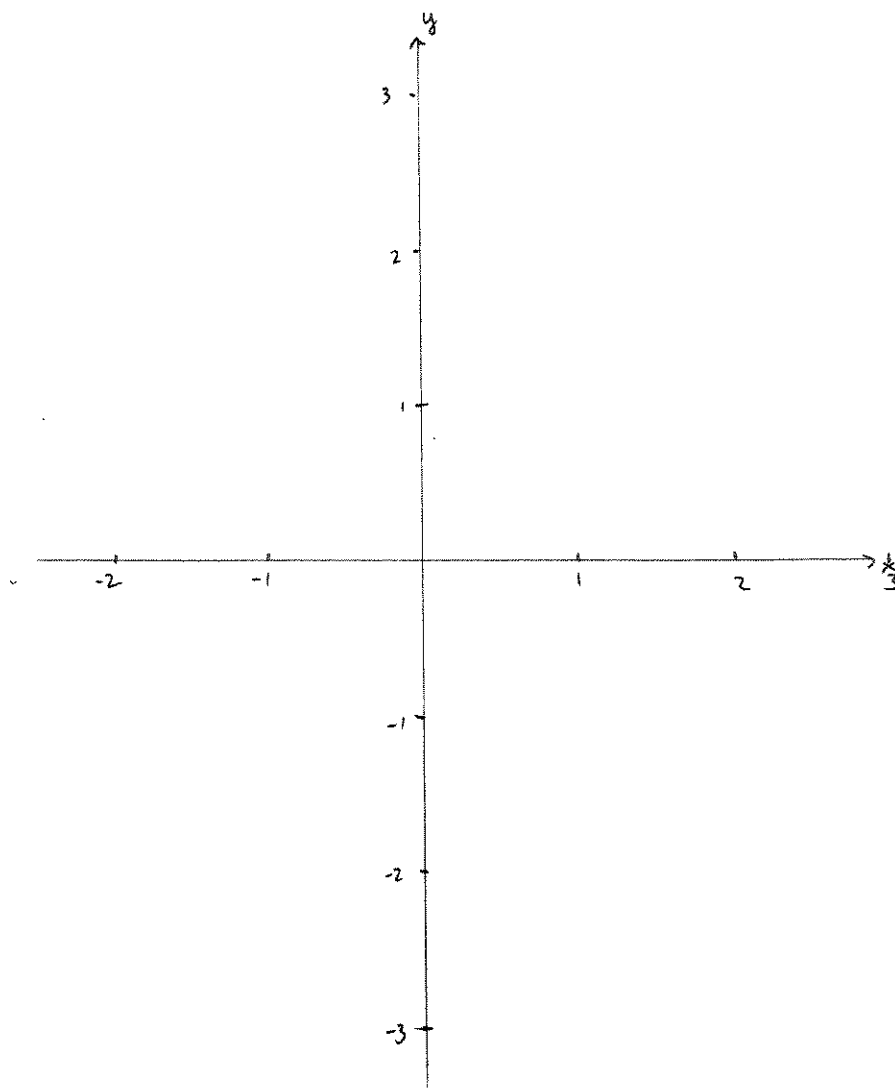
example 2

Make the slope field picture
for example 2,

$$\frac{dy}{dx} = y - x$$

Then sketch the IVP solution
onto the slope field

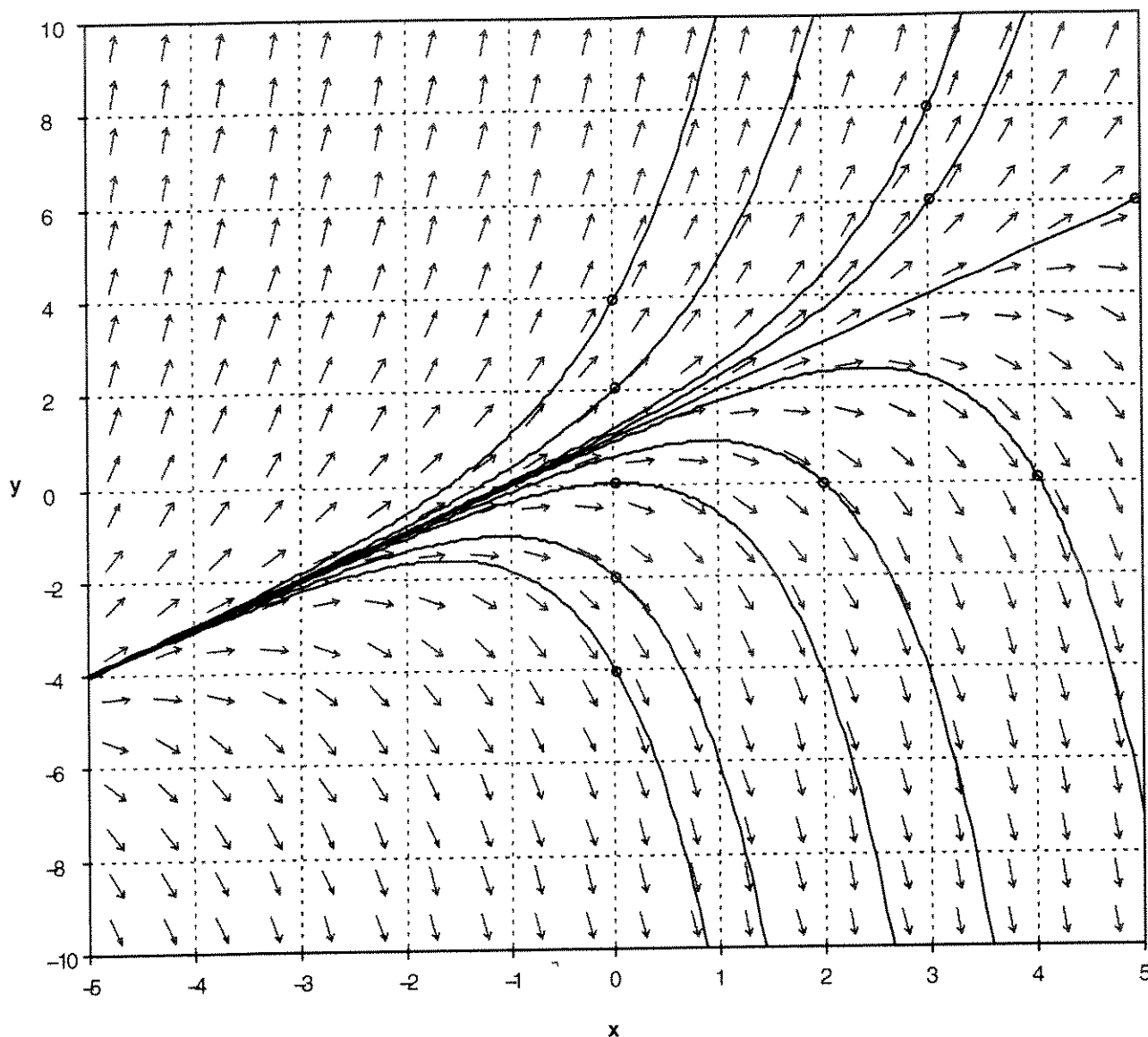
slope value	isocline equation
0	
1	
-1	
2	
-2	



you can do this with computer help too:

(4)

$$y' = y - x$$



google "dfield" to get an applet that lets you have the computer create direction fields. Then, if you click the mouse on an initial point (e.g. $(0,0)$ for IVP $y(0)=0$), the applet will numerically follow the slope field backwards & forwards in the horizontal variable, and graph the sol'n field. (Math Dept printing problems with this applet.)

in the Math Dept, in a terminal window, type after the prompt,

dfield &

to open a unix version that prints.

↪ <http://math.rice.edu/dfpp.html>