Math 2280-1
MTWF 8:35-9:25
LCB 215

Monday
Jan. 11

HW for this Friday Jan. 15
hand in circled problems; show steps.
uncircled problems are optional.
A subset of the circled problems will be graded.

1.1 3, 6 15 16 19 27, 31 34 35 36
1.2 6, 7, 13 18 19, 20 25, 29 33 34 39

- syllabus

- what is a differential equation?

  - n^{th} order DE: $F(x, y, y', y'', \ldots y^{(n)}) = 0$

    i.e. an equation involving a variable "x" and an (unknown) function $y(x)$ & its first n derivatives

- where do differential eqtns come from?

  - mathematical models, often of continuous dynamical systems in which the variable "x" is actually time "t".

- goal:

  - understand the "solution function(s)" $y(x)$, i.e. the functions for which the differential equation is true.

<u>Examples</u>:

(1) In calculus you have seen

$$\frac{dP}{dt} = kP \qquad \begin{array}{l} \text{"}x\text{"} = t \\ \text{"}y(x)\text{"} = P(t) \end{array}$$

$k>0$: population growth rate is proportional to population
$k<0$: " decay " " " "

Solve this DE!

<u>Chain rule backwards</u>

same as <u>differentials</u> (separable DE.)

(2) Newton's Law of cooling

$A =$ ambient temp.

↑ object with temp $T(t)$

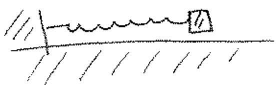
$$\frac{dT}{dt} = k(A-T)$$

"rate of change of temp is proportional to difference between temp and ambient temp"

(3) Spring-mass configuration

m = mass

wall



flat table

$x(t)$ = mass displacement from equilibrium at time t

2^{nd} order DE, from Newton's Law and "linearization"

$$\underset{\text{mass}\cdot\text{acceleration}}{m\,x''(t)} = \text{net forces on mass} = -kx - cx'$$

If you assume force depends only on position x and velocity x', $F = F(x, x')$ and $F(0,0) = 0$ (no net forces at equilibrium) then this is the linear part of the Taylor series for F, i.e. we ignore higher order terms.

$k :=$ Hooke's const

$c :=$ coef of drag.

Example : Consider the no-drag case,

$*$ $\quad x''(t) + 16\,x(t) = 0.$

- Show $x(t) = A\cos 4t + B\sin 4t$ solves this D.E.
- Solve the initial value problem for $*$, with initial displacement $= 1$, initial velocity $= 2$

(4) We ended Math 2270 discussing <u>discrete</u> <u>dynamical</u> <u>systems</u>, in particular ones of the form

$$\begin{cases} \vec{x}(t+1) = A\,\vec{x}(t) \\ \vec{x}(0) = \vec{x}_0 \end{cases}$$

e.g. Coyote - roadrunner
Glucose - insulin
input - output models
Google voting game

These discrete dynamical systems are related to ODE's we shall study (starting in <u>chapter 4</u>)...

$$\vec{x}(t+1) = A\,\vec{x}(t)$$

iff $$\frac{\vec{x}(t+1) - \vec{x}(t)}{1} = (A-I)\,\vec{x}(t)$$

iff $$\frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t} = B\,\vec{x}(t) \qquad (B = A-I,\ \Delta t = 1)$$

looks like the <u>first</u> <u>order</u> <u>system</u> of <u>DE's</u>

$$\begin{cases} \vec{x}\,'(t) = B\,\vec{x}(t) \\ \vec{x}(0) = \vec{x}_0 \end{cases}$$

(2) Newton's Law of cooling

murder mystery... a body is found at 3 A.M. temp = 85°

by 4 A.M. temp = 80°

$65^\circ = A$

about when did the murder occur?

$$\frac{dT}{dt} = k(A - T) \ \ldots$$