Math 2280-1
Friday Feb. 26

HW for Fri March 5

3.6 7, 13, 16, 20, 21, 22

3.7 2, 3, 4, 7, 11

4.1 1, 8, 11, 15, 16, 21, 24, 26

4.3 9 You may hand in joint work with up to 1 other person on this problem, which will require you to write some Runge-Kutta code for systems. (You can help as many people as you want!)

Class problems A, B in today's notes.

- finish the transition from "beating" to "resonance" in undamped forced oscillations
  Wed - notes p. 2.5—3

- discuss the conservation of energy method for deriving eqtns of motion and natural angular frequency, + washboarding example on p. 4 Wed.

you may also want to recall how moments of inertia are related to rotational KE via multiple integration, e.g.

moment of inertia for rotating disk

$$KE = \iint \frac{1}{2}(\overset{v^2}{\overbrace{(r\omega)^2}})\,\overset{dm}{\overbrace{\rho\, r\,dr\,d\theta}} \qquad \rho = \frac{M}{\pi a^2}$$

$$= \frac{1}{2}\rho\omega^2 \underbrace{\int_0^{2\pi} \underbrace{\int_0^a r^3\,dr}_{\frac{a^4}{4}}\,d\theta}_{\frac{\pi a^4}{2}}$$

$$= \cancel{\frac{1}{4}\pi \frac{M}{\pi a^2}}$$

$$= (\rho\pi a^2)\,\frac{1}{4}(a\omega)^2$$

$$= \frac{1}{4} M a^2 \omega^2$$

$$= \frac{1}{2} I \omega^2 \quad \text{for } \boxed{I = \frac{1}{2} M a^2.}$$

Case 3 Forced oscillations with damping.

example 6 p. 220: Solve

$$\begin{cases} x'' + 2x' + 26x = 82\cos 4t \\ x(0) = 6 \\ x'(0) = 0 \end{cases}$$

$x_H(t)$:

$x_p(t)$:

IVP:

question to fool you:

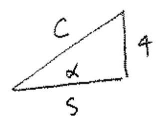
what form would your guess for $x_p(t)$ take if the RHS was $82\cos 5t$?

orig prob.
<u>ans:</u>

$$x(t) = \underbrace{5\cos 4t + 4\sin 4t}_{\substack{C\cos(4t-\alpha);\\ \boxed{x_{sp}(t)}}} + \underbrace{e^{-t}(\cos 5t - 3\sin 5t)}_{\boxed{x_{tr}(t)}}$$

$C = \sqrt{41}$

$\alpha = \arctan(.8)$

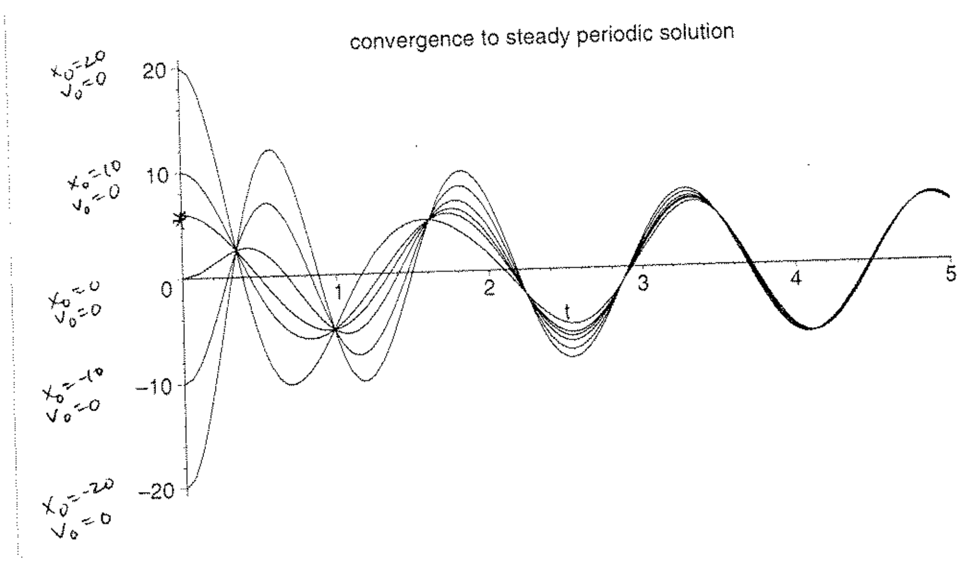


the <u>steady</u> <u>periodic</u> part of the solution, is independent of initial conditions

(the <u>transient</u> part of the soltn; decays as $t \to \infty$)

$x_{tr}(t)$ is determined by the initial conditions

see figure 3.6.8 p. 22[redacted], or below



general case: (for reference)

$$m x'' + c x' + k x = F_0 \cos\omega t \qquad c \neq 0$$

$$\begin{aligned}
k(\quad & x_p = A\cos\omega t + B\sin\omega t \\
+\ c(\quad & x_p' = -A\omega \sin\omega t + B\omega\cos\omega t \\
+\ m(\quad & x_p'' = -A\omega^2\cos\omega t - B\omega^2 \sin\omega t
\end{aligned}$$

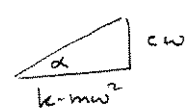
$$L(x_p) = \cos\omega t \left[A(k - m\omega^2) + Bc\omega\right] + \sin\omega t\left[A(-c\omega) + B(k-m\omega^2)\right] = \cos\omega t\,[F_0] + \sin\omega t\,[0]$$

$$\begin{bmatrix} k-m\omega^2 & c\omega \\ -c\omega & k-m\omega^2 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{(k-m\omega^2)^2 + c^2\omega^2}\begin{bmatrix} k-m\omega^2 & -c\omega \\ c\omega & k-m\omega^2 \end{bmatrix}\begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{F_0}{(k-m\omega^2)^2 + c^2\omega^2}\begin{bmatrix} k-m\omega^2 \\ c\omega \end{bmatrix}$$

so $x_p(t) = C\cos(\omega t - \alpha)$



$$C = \sqrt{A^2 + B^2}$$

$$C = \frac{F_0}{(k-m\omega^2)^2 + c^2\omega^2}\sqrt{(k-m\omega^2)^2 + c^2\omega^2}$$

$$\boxed{C = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}, \quad \alpha = \arccos\left(\frac{k-m\omega^2}{C}\right)}$$

notice denom can be very small if
$\omega \approx \omega_0$ (makes 1st term small)
$c \approx 0$ (makes second term small). } this makes $C \gg F_0$ "practical resonance"

then

$$x(t) = \underbrace{C\cos(\omega t - \alpha)}_{x_{sp}(t)} + \underbrace{x_H(t)}_{x_{tr}(t)} = \begin{cases} c_1 e^{-r_1 t} + c_2 e^{-r_2 t} & \text{overdamped} \\ e^{-rt}(c_1 + c_2 t) & \text{crit damped} \\ e^{-at}(A\cos bt + B\sin bt) & \text{underdamped.} \end{cases}$$

practical resonance occurs when C is $\gg F_0$, (you decide.)

i.e. when $\omega \approx \omega_0$, $c \approx 0$.

example :

$$x'' + 2x' + 26x = F_0 \cos \omega t$$

plot the fcn

$$\frac{1}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}}, \text{ as a fcn of } \omega \quad \left(\begin{array}{l} m = 1 \\ k = 26 \\ c = 2 \end{array} \right.$$

$$= \frac{1}{\sqrt{(26 - \omega^2)^2 + 4\omega^2}}$$

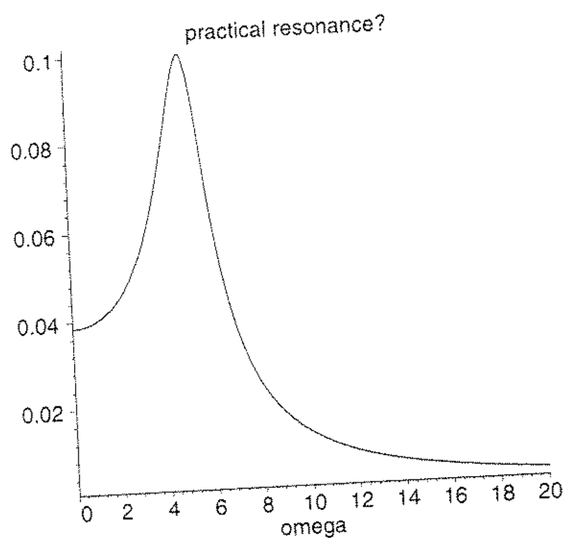


figure 3.6.9 p.221

$\omega_0 = \sqrt{26} \approx 5$