Math 2280-1
Friday April 9

HW for Fri April 16 (1)

6.3 8 9 10, 14 15, 16, 17
6.4 12 13 14 15 16

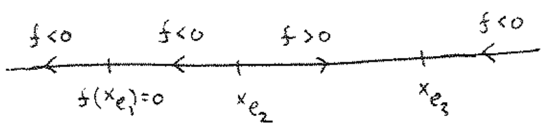
in 6.3.8 and 6.3.10 create a pplane phase portrait for the nonlinear system (3), and explain how your linearization computations are reflected in the non-linear behavior near the corresponding equilibria

? 7.1 3 7 8 13, 20, 21, 23, 28
7.2 3, 4 5 6, 14 19 20, 28 31.

- Finish analysis of competition model, p. 3-6 on Tuesday notes.

- Overview of how solution complexity increases as the number of autonomous 1^{st} order DE's in a system increases

$\underline{n=1}$ $\begin{cases} \frac{dx}{dt} = f(x) \\ x(0) = x_0 \end{cases}$



<u>Theorem</u> Let $x(t)$ solve IVP. Then either

(a) $\lim_{t\to\infty} x(t) = \pm\infty$

or (b) $\lim_{t\to\infty} x(t) = x_e$, an equil soltn.

(analogous statement for $t \to -\infty$)

and we could prove this!

$\underline{n=2}$ $\begin{cases} \frac{dx}{dt} = f(x,y) \\ \frac{dy}{dt} = g(x,y) \\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \end{cases}$

<u>Theorem</u> Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ solve IVP. Then either

(a) $\lim_{t\to\infty} \left\| \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \right\| = \infty$

(b) $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_e \\ y_e \end{bmatrix}$ an equilibrium soltn

<u>or</u>

new possibility! → (c) There exists a periodic soltn ("cycle") $\begin{bmatrix} \tilde{x}(t) \\ \tilde{y}(t) \end{bmatrix}$ so that $\lim_{t\to\infty} \left\| \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} - \begin{bmatrix} \tilde{x}(t) \\ \tilde{y}(t) \end{bmatrix} \right\| = 0.$

(this is an interesting theorem to prove, early 1900's, Birkhoff)

$\underline{n=3}$ $(n \geq 3)$: (a), (b), (c), or even "chaotic" behavior, and "strange attractors" §6.5

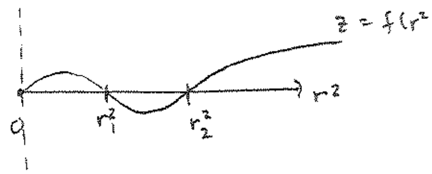
<u>Note</u>: a system $\frac{d\vec{x}}{dt} = \vec{F}(t, \vec{x})$ of k 1^{st} order DE's is equivalent to an autonomous system of $(k+1)$ 1^{st} order DE's!

$\begin{cases} \frac{d\vec{x}}{dt} = \vec{F}(\tau, \vec{x}) \\ \frac{d\tau}{dt} = \tau \end{cases}$, so e.g. a 2^{nd} order forced oscillation eqtn is like a 1^{st} order autonomous system of 3 DE's. ~ such systems can exhibit chaos

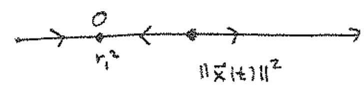
- <u>Examples</u> with possibility (c)

$$* \quad \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -y \\ x \end{bmatrix}}_{\text{Rot}_{\pi/2}\begin{bmatrix} x \\ y \end{bmatrix}} + \underbrace{f(r^2)\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{radial field}} \qquad r^2 = x^2+y^2$$

this linear system has circular trajectories



circular orbits at r_1 & r_2.



for soltns to $*$, $\left(\frac{1}{2}(x^2+y^2)\right)' = xx' + yy'$

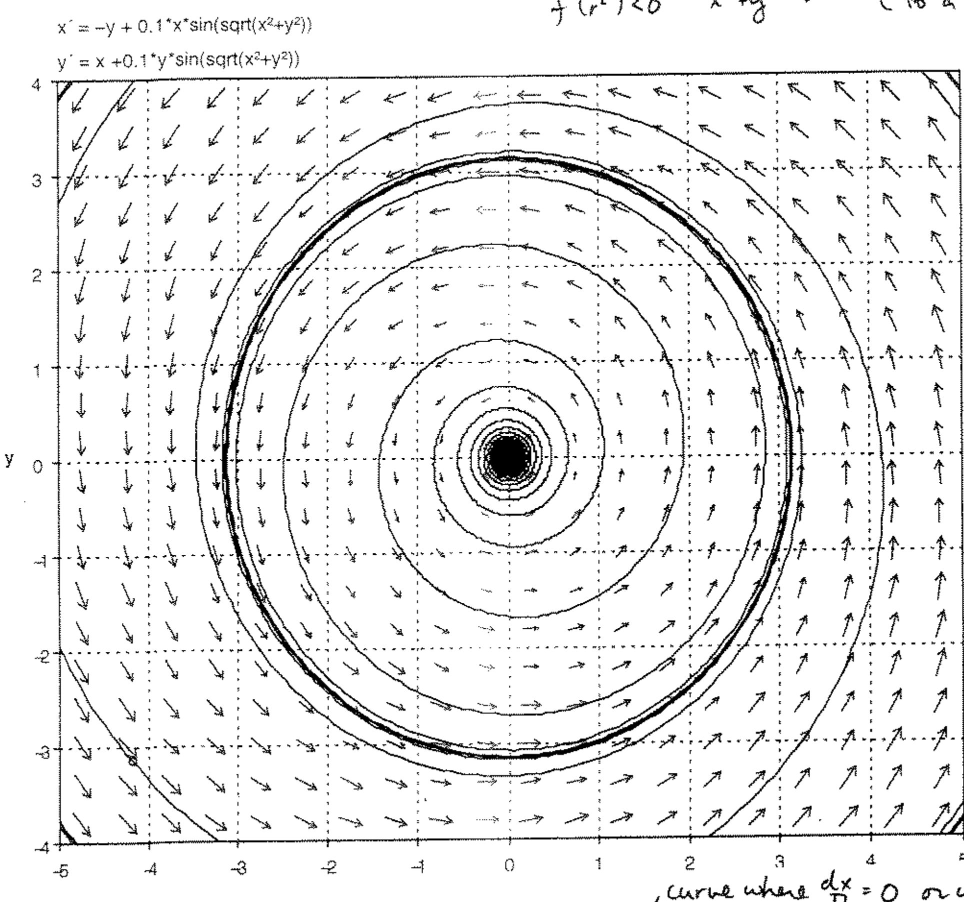
$$= x(\cancel{-y} + f(r^2)x) + y(\cancel{x} + f(r^2)y)$$

$$= f(r^2)(x^2+y^2)$$

so if $f(r^2)=0$, x^2+y^2 is const

$f(r^2)>0$ $\quad x^2+y^2 \nearrow$ (to a zero of f, or to ∞)

$f(r^2)<0$ $\quad x^2+y^2 \searrow$ (to a zero of f, or to 0)



curve where $\frac{dx}{dt}=0$ or where $\frac{dy}{dt}=0$

nullclines intersect at equilibria

- $\boxed{\text{play with pplane}}$. Discuss "seperatrix", "nullcline", "stable & unstable orbits"

solutions converging to saddle pt equilibria, either as $t \to \infty$ or $t \to -\infty$