Math 2280
Tuesday April 27
9.6 Waves

- on page 2 of Monday notes, don't forget that we corrected the product fcn solns to heat eqn

$$u_t = k u_{xx}$$

solns $(\sin wx)e^{-kw^2 t}$, $(\cos wx)e^{-kw^2 t}$ originally were missing their diffusivity constants k

so for a type (1) problem
$(u(0,t) = u(L,t) = 0 \ \forall t)$
we can write the initial temp

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{\pi}{L} n x\right)$$

and soln $$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{\pi}{L} n x\right) e^{-k\left(\frac{\pi}{L} n\right)^2 t}$$

analogously for a type (2) problem
$(u_x(0,t) = u_x(L,t) = 0 \ \forall t)$
write init. temp

$$f(x) = \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi}{L} n x\right) + \frac{a_0}{2}$$

& soln $$u(x,t) = \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi}{L} n x\right) e^{-k\left(\frac{\pi}{L} n\right)^2 t} + \frac{a_0}{2}$$

- On page 6 Monday, which we'll discuss today, the type (2) building block (2^{nd} fcn on the right side) for free ends with only an initial velocity fcn, should be

$$\cancel{\sin} \cos\left(\frac{n\pi x}{L}\right) \sin\left(\frac{a n \pi}{L} t\right)$$, lower right.

↑
notes say
$\sin\left(\frac{n\pi x}{L}\right)$

- Discuss wave eqn p. 4-6 Monday, see Maple visualizations (linked at our home page) and do slinky math, p. 2

Slinky math

assume equil. slinky has length ≈ 0, and is Hooke's-like
and mass m.
So if it's stretched to length L

$$\rho = \frac{m}{L}$$

$$T = kL \qquad k = \text{Hooke's const.}$$

transverse oscillations

$$a = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{kL}{m/L}} = L\sqrt{\frac{k}{m}}$$

parallel oscillations:

$$T = f(\rho) = kL = \frac{km}{\rho}$$

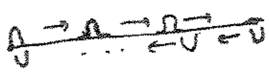
$$T'(\rho) = -\frac{km}{\rho^2}$$

$$a = \sqrt{-T'(\rho)} = \sqrt{\frac{km}{\rho^2}} = \sqrt{\frac{km}{m^2/L^2}} = L\sqrt{\frac{k}{m}}$$

!!

1 test this

for our slinky, $m = .2268$ kg
and there are 88 loops
we can measure k if we want.

pulse period:  ← fixed endpoint
waves return upside down.
Why?

time for a wave to
go back and forth
(i.e. distance $2L$) is

$$(\text{speed})(\text{time}) = 2L$$

$$\text{time} = \frac{2L}{L\sqrt{k/m}} = 2\sqrt{\frac{m}{k}}$$ is independant of L !!

2 test this

alternate way to measure speed:

fundamental mode (fixed endpoint)

$$y(x,t) = \sin\left(\frac{x\pi}{L}\right)\cos\left(\frac{a\pi}{L}t\right)$$ same a!

$$\left(y = \frac{1}{2}\left[\sin\left(\frac{\pi}{L}(x-at)\right) + \sin\left(\frac{\pi}{L}(x+at)\right)\right]\right)$$

$$\text{one period} = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{a\pi}{L}\right)}$$

$$= \frac{2L}{a} = \frac{2L}{L\sqrt{k/m}} = 2\sqrt{\frac{m}{k}}$$

= pulse period

3 test this