

Sinusoidal function essentials
(for first Maple project & later, see §3.4)

These notes explain how to rewrite the linear combo

$$F(t) = A \cos \omega t + B \sin \omega t$$

in amplitude phase form

$$F(t) = C \cos(\omega t - \alpha) \\ = C \cos(\omega(t - \delta))$$

$$C > 0$$

$$\alpha = \omega \delta, \quad \delta = \alpha / \omega$$

C = amplitude

ω = angular frequency

radians/time

α = phase angle

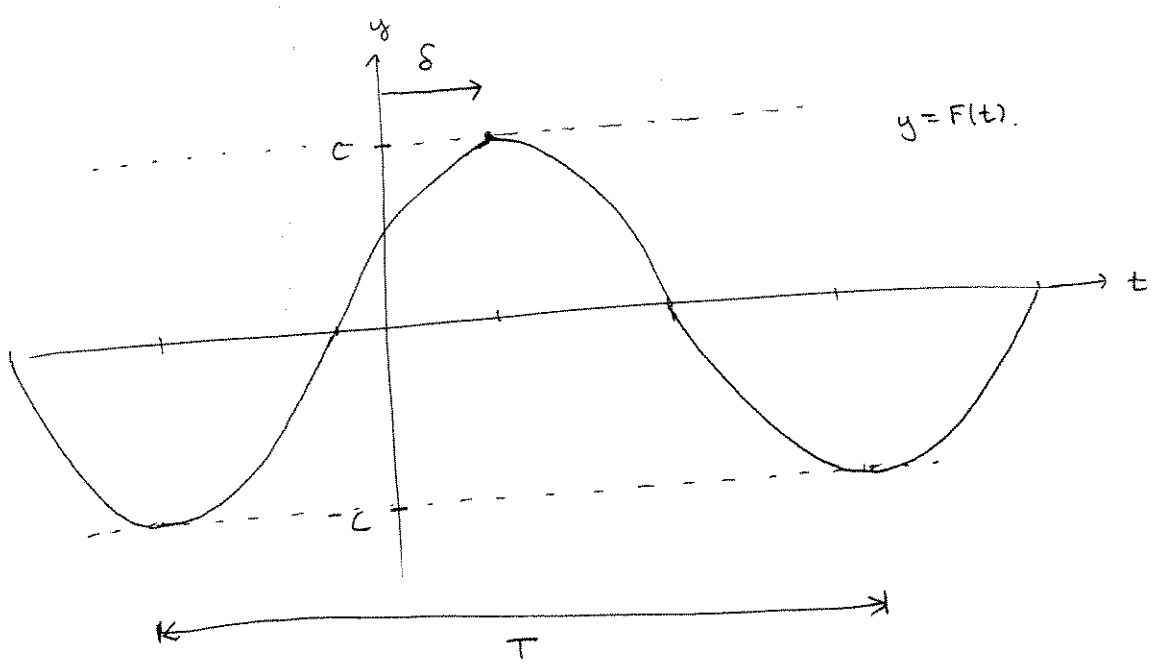
δ = time delay

~ graph of F is graph of $C \cos \omega t$ translated δ units to the right.

$$f = \text{frequency} = \frac{\omega}{2\pi} \text{ cycles/sec}$$

$$T = \text{period} = 1/f = \frac{2\pi}{\omega} \text{ sec/cycle}$$

(or other time unit).



Recall $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$

Thus

$$C \cos(\omega t - \alpha) = C \cos\omega t \cos\alpha + C \sin\omega t \sin\alpha$$

$$\stackrel{?}{=} A \cos\omega t + B \sin\omega t \quad \text{as fns of } t$$

possible iff

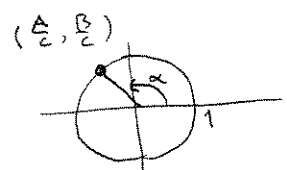
$$A = C \cos\alpha$$

$$B = C \sin\alpha$$

iff

$$\frac{A}{C} = \cos\alpha$$

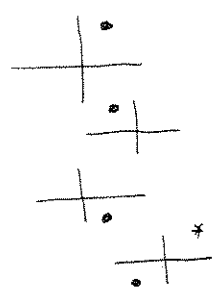
$$\frac{B}{C} = \sin\alpha$$



there exists such an angle iff

$$\left(\frac{A}{C}\right)^2 + \left(\frac{B}{C}\right)^2 = 1 \quad \text{i.e.} \quad \boxed{C = \sqrt{A^2 + B^2}}$$

and then use inverse trig to find α



- $(A, B) \in 1^{\text{st}}$ quadrant
- $(A, B) \in 2^{\text{nd}}$ " "
- $(A, B) \in 4^{\text{th}}$ " "
- $(A, B) \in 3^{\text{rd}}$ " "

$$\alpha = \arctan \frac{B}{A} = \arcsin\left(\frac{B}{C}\right) = \arccos\left(\frac{A}{C}\right)$$

$$\alpha = \arccos\left(\frac{A}{C}\right)$$

$$\alpha = \arctan\left(\frac{B}{A}\right) = \arcsin\left(\frac{B}{C}\right)$$

$$\alpha = \arctan\left(\frac{B}{A}\right) + \pi$$

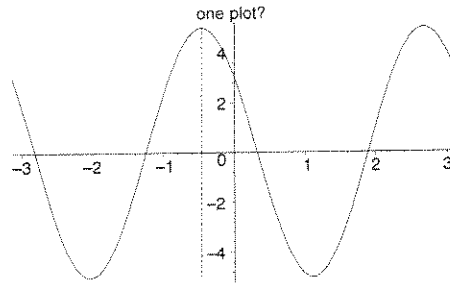
Example Express $F(t) = 3 \cos 2t - 4 \sin 2t$ in phase amplitude form

ans

$$F(t) = 5 \cos(2t + .927)$$

$$= 5 \cos(2t + (.927 - 2\pi))$$

```
> with(plots):  
plot1:=plot(3*cos(2*t)-4*sin(2*t),t=-Pi..Pi,color=black):  
plot2:=plot(5*cos(2*t+.927),t=-Pi..Pi,color=red):  
plot3:=plot([-1.46,t,t=-5..5],linestyle=2,color=black):  
display({plot1,plot2,plot3},title='one plot?');
```



```
> ?plot #help window to help me figure out to draw  
# the dotted line above, to show graph translation
```