

Math 2280-1
Maple Project 1
Population models, section 2.1
Numerical Methods, sections 2.4-2.6
Problems due Friday February 3

Create a Maple document, perhaps by modifying this one, in which you combine text and Maple commands to answer (and explain) the following problems:

(0) Be sure to include your name(s) at the top of your document. You may work individually or in groups of up to three people, and each group should hand in only one document.

Populations:

(1) Complete Investigation C on page 89 of the text. This involves finding parameters for a logistic model of world population from Figure 2.1.10 on page 90, and then predicting world population in 2025. We carried out an analogous study for U.S. populations, and the Maple file located at <http://www.math.utah.edu/~korevaar/2280spring06/jan20maple.mws> has useful commands to cut, paste, modify.

Numerical techniques:

(2) Consider the same initial value problem as in the class handout <http://www.math.utah.edu/~korevaar/2280spring06/numerical1.mws>, namely

$$\frac{dy}{dx} = y$$
$$y(0) = 1.$$

Use the improved Euler algorithm, with $n = 100$, on the x -interval $[0,1]$, to approximate the solution. Print out your approximations at x values which are multiples of 0.1 . Compare the accuracy of the $n=100$ improved Euler approximation to e , versus the results in the handout for unimproved Euler. Note, that on this problem and the subsequent ones, the class handout has useful routines to copy, paste, and modify.

(3) Use the Runge-Kutta algorithm for the same initial value problem and x -interval as in (2) above, with $n=20$. Compare the accuracy of your results to the work you did there using improved Euler $n=100$. There should be a huge improvement.

(4) Do problem #29 in Section 2.4, page 120. (This problem involves work done by hand in part 29a. You may staple this work to your Maple file if you don't want to bother typing it. But use Maple for 29bc) This problem illustrates how a numerical algorithm may accidentally pass right through an x -value where the real solution blows up.

(5) Do problem #5 in sections 2.4, 2.5, 2.6 homework. This is the same initial value problem in each case, studied successively with Euler, improved Euler, and Runge-Kutta.

(6) Do problem #25 in section 2.5. Additionally, compare your numerical answers to the exact answers obtained by solving this initial value problem by hand. (As in problem 4 above, you may either staple

your hand-work to the Maple file or type it directly in.)

(7) Go to example 4 on page 138-139. Work through and understand what the text has to say on these two pages. Recreate the data in the last two columns of Figure 2.6.9 on page 138 using the Runge-Kutta routine from the class handout. That is, recompute the the Runge-Kutta approximations with $h=0.05$ and compare to the actual solution values, at multiples of 0.4.

Understand (for yourself) how the form of the general solution

$$y(x) = e^{(-x)} + C e^{(5x)}$$

means that small variations in initial values y_0 lead to huge variations in $y(x)$ for $x \gg 0$. This sort of instability in actual solution behavior leads to numerical instability like that illustrated in this important example, even when you're using a "good" numerical technique like Runge Kutta and a "small" time step. Thus people who create numerical solutions to problems must also be well-grounded in the mathematical theory behind the problems.