Math 2280-1
Tuesday 24 Jan

Remember we meet in the computer classroom (LCB 115, next door) tomorrow (Wed 25 Jan)!

Recall these notions:

<u>autonomous</u> first order differential equation

<u>equilibrium solution</u> (to aut DE)

<u>stable/unstable</u> equilibrium

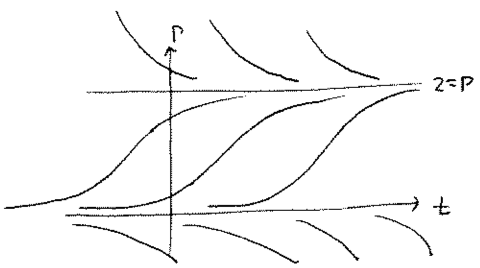
- Doomsday/extinction model on page 4 of Monday notes

- Harvesting a logistic population (e.g. fisheries)

$$\frac{dP}{dt} = \underbrace{aP - bP^2}_{\text{logistic}} \underbrace{- c}_{}$$

constant rate harvesting. (see §2.2 #24)
one could also consider a term $-cP$, which would maybe correspond to constant effort harvesting (see §2.2 #23)

e.g. $\frac{dP}{dt} = 2P - P^2$
$= P(2-P)$



vs $\frac{dP}{dt} = 2P - P^2 - c$

consider what happens for different c values

roots of RHS: $P^2 - 2P + c = 0$ has roots

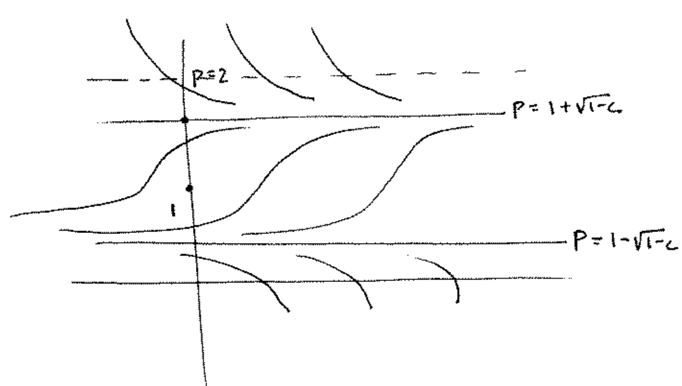
$$P = \frac{2 \pm \sqrt{4-4c}}{2} = 1 \pm \sqrt{1-c}$$

so, for $0 \le c \le 1$,

$$\frac{dP}{dt} = -\left(P - (1+\sqrt{1-c})\right)\left(P - (1-\sqrt{1-c})\right)$$

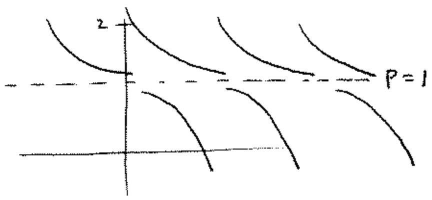
still looks logistic (& fishery won't collapse) if $c \ll 1$

extinction zone

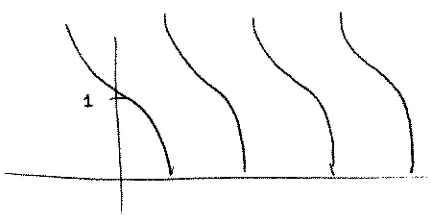


as $c \nearrow 1^-$ the middle zone (where populations increase to the modified carrying capacity) shrinks in width, until at $c = 1$ the two roots coalesce;

$$\frac{dP}{dt} = 2P - P^2 - 1 = -(P^2 - 2P + 1) = -(P-1)^2$$



and for $c > 1$, $\quad 2P - P^2 - c = -(P^2 - 2P + c) = -((P-1)^2 + (c-1)) < 0 \quad \forall P$ (but least negative @ $P = 1$)



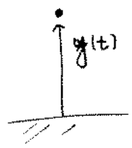
extinction $\forall P_0$!

this model gives a plausible explanation for why fishery after fishery has collapsed around the world – if $c < 1$ but near 1, and if something perturbs the system a little bit (e.g. increased fishing pressure, a big storm, etc.), you could be confronted with "sudden", unexpected fishery collapse.

§2.3 Improved models of projectile motion

(but still vertical motion v = velocity, $\frac{dy}{dt}$)

<u>linear model</u>



$$m\frac{dv}{dt} = F_R + F_G = -kv - mg$$

F_R: air resistance force

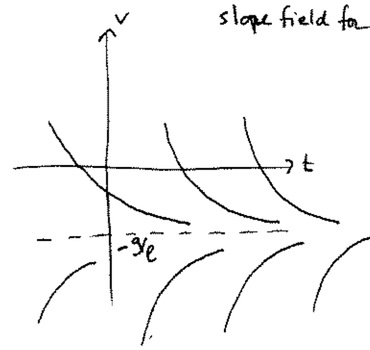
F_G: gravity force

yields $\quad \frac{dv}{dt} = -\rho v - g \qquad (\rho = k/m)$

$$= -\rho\left(v + \frac{g}{\rho}\right)$$

solve either by separating variables, or as a linear DE

slope field for velocity



we will do examples with the linear model, and also study a non-linear drag (quadratic) model,

$$m\frac{dv}{dt} = -mg - kv|v|$$

<u>ans</u>: $v = \underbrace{-g/\rho}_{:= v_T\text{; terminal velocity}} + (v_0 + g/\rho)e^{-\rho t}$

$$y = y_0 + v_T t + \frac{1}{\rho}(v_0 - v_T)(1 - e^{-\rho t})$$