Math 2280-1
Tuesday Jan 17

Refer to picture on Friday notes
for Torricelli picture

Recall, we used Physics to deduce that
the speed water leaves the ~~system~~ cistern is

$v = \sqrt{2gy}$ where $y(t)$ is the water level above the cistern hole.

alternate equalities

$\frac{dVol}{dt} = A(y)\frac{dy}{dt}$

$\frac{dVol}{dt} = -va$

This leads to a separable DE for $y(t)$:

In time dt, $dVol = A(y)\,dy$; $A(y)$ = sectional area at height y
note $dy<0$, $dVol<0$

but also $dVol = (-v\,dt)\,a$; a = sectional area of hole

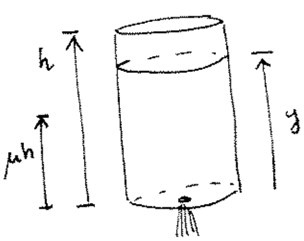
Thus
$$A(y)\,dy = -va\,dt \underset{\uparrow \text{ Torricelli}}{=} -a\sqrt{2gy}\,dt$$

$$\boxed{A(y)\frac{dy}{dt} = -a\sqrt{2g}\,y^{1/2} = -k\,y^{1/2}}$$

(some minus signs were missing on left side of page 1 Fri)

__Example__ Cylindrical cistern: $A(y) = A$ const.

$$\frac{dy}{dt} = -k\,y^{1/2} \quad (\text{different } k).$$



Let T be the amount of time it takes the cistern to empty from full height h, to height μh $(0<\mu<1)$
Show the time it takes to empty the tank is given by

$$\boxed{T_{tot} = \frac{T}{1-\mu^{1/2}}}$$

(there's room on the next page to check this)

Nalgene bottle experiment:

I marked off the bottle so that we can use $\mu = .5$

So if we time how long it takes to empty half the height, and call it T,

then the total time estimate will be

$$T_{tot} = \frac{1}{1-\sqrt{.5}}\, T \approx (3.41)\, T$$

Experiment

$T =$

$(3.41)T =$

$T_{tot} =$

§ 1.5 Linear 1st order DE's.

$$(1) \qquad \frac{dy}{dx} + P(x)y = Q(x)$$

Notice the left side of this eqtn, $L(y) := y' + P(x)y$ is <u>linear</u> :

$$L(y_1 + y_2) = L(y_1) + L(y_2) \qquad (y_1, y_2 \text{ diff'ble})$$
$$L(cy) = cL(y) \qquad (c \text{ a const, } y \text{ diff'ble})$$

Solution method is to multiply both sides by a non-zero fcn ("integrating factor") so that we can antidifferentiate wrt x to deduce $y(x)$:

eqtn (1) is equiv. to

$$(2) \qquad e^{\int P(x)dx}\left(y' + P(x)y\right) = e^{\int P(x)dx} Q(x)$$

where $\int P(x)dx$ is any particular antideriv. of $P(x)$

equiv to

$$(3) \qquad \frac{d}{dx}\left[e^{\int P(x)dx} y\right] = e^{\int P(x)dx} Q(x)$$

or (antidifferentiating)

$$(4) \qquad e^{\int P(x)dx} y = \int [\quad] \, dx$$

; divide by $e^{\int P(x)dx}$ to get $y(x)$

<u>Example</u> HW problem #6 §1.3 you are asked to plug in $y = x + Ce^{-x}$ to show it solves the DE

$$y' = x - y + 1$$

Use the algorithm above to <u>derive</u> the given solns :

$$y' + 1y = x + 1$$

example 2 page 48-49:

$$(x^2+1)\frac{dy}{dx} + 3xy = 6x$$

ans $y = 2 + C(x^2+1)^{-3/2}$

[redacted]