Math 2280-1
Wed 11/11

* finish summer problem from Tuesday.

- separable DE's
- y' = 3 - 1.4
  slope fields; existence & uniqueness for solutions to 1st order IVP.
  "E" "I""slope field idea indicates that we may be able to solve 1st order IVP's, uniquely."

Example 1
\[ \frac{dy}{dx} = -3y \]

- draw slope field, and DE solution graphs y = y(x).
- solve the DE by separating variables, compare to slope field picture
- explain why every IVP has a unique solution, formally

optional problem session Thurs, 11/13, 9:40-10:30
this week's hw (assigned Monday), due Fri.
You'll continue to work on #1
Next week's hw will be given on Fri.
Part B it will be

\[ y(1) = 3, 6, 10, 11, 12, 13, 18, 21, 23, 29 \]

also verify
\[ y(x) = x + ce^{-x} \]
solve the DE and convince yourself the curves you sketch on the slope field are consistent with this.

Explain why
Every thing fails to apply, but solve this separable DE to show the IVP has solutions.
a separable DE can be written as
\[
\frac{dy}{dx} = \frac{f(x)g(y)}{g(y)}
\]
and solved by the algorithm
\[
g(y) \frac{dy}{dx} = f(x) dx
\]
\[
\int g(y) \, dy = \int f(x) \, dx
\]
where \(G(y) = \int g(y) \, dy\) is antiderivative of \(g(y)\)
\(F(x) = \int f(x) \, dx\) is antiderivative of \(f(x)\)

\[
G(y) = F(x) + C
\]
expresses \(y\) implicitly as a function of \(x\).

You may be able to solve this equation explicitly for \(y = y(x)\).

Example 2:
\[
\frac{dy}{dx} = 1 + y^2
\]

- draw pictures of solution graphs on to the slope field. In particular, try
- find solutions by separating variables
- explain why every IVP has a solution, but this solution does not exist for all \(x\)

Example 2:
> with(DEtools):
> #Maple tools for differential equations
> deqtn:=diff(y(x),x)=1+y(x)^2;
> #Example 1
> deqtn := \frac{dy}{dx}(y(x))=1+y(x)^2
> dfieldplot(deqtn, y(x),x=-3..3,y=-3..3, arrows=line, color=black, dirgrid=[30,30]);
Just for fun:

```maple
> DEplot(deqtn, y(x), x=-3..3, [[y(0)=0], [y(1)=0], [y(-1)=0]], y=-3..3, arrows=line, color=black, linecolor=black, dirgrid=[30,30]);
```

---

**Example 3**

\[
\begin{align*}
\frac{dy}{dx} &= y^3 \\
y(0) &= 0
\end{align*}
\]

Using the slope field (Existence holds, uniqueness fails!)

- Find the graphs of lots of solutions to this IVP
- Solve, using separation of variables, and articulate with 1st bullet point

```maple
> deqtn:=diff(y(x),x)=abs(y(x))^(2/3);
> fieldplot(deqtn, y(x), x=-2..2, y=-1..1, arrows=line, color=black, dirgrid=[40,20]);
```

```
deqn := \frac{dy}{dx} = |y(x)|^{2/3}
```
Usually, existence and uniqueness hold.

**Theorem 1** ∃! : (page 23)

Consider

\[
\begin{align*}
\text{IVP} & \quad \frac{dy}{dx} = f(x,y) \\
y(a) &= b
\end{align*}
\]

Let \((a,b)\) be interior to a closed rectangle \(R\)

\(a_1 \leq x \leq a_2, \ b_1 \leq y \leq b_2\)

Let \(f(x,y)\) be continuous in \(R\).

Then \(\exists\) sol'n to IVP on some interval \(J\)
containing \(a\) in its interior

If \(\frac{df}{dy}\) exists and is continuous in \(R\), then this
solution is unique on any subinterval \(J_0\)
s.t. the solution graph lies inside \(R\).

- See how this theorem applies to the 3 examples.
- The proof of theorem 1 is complicated and is indicated in an Appendix.
  It uses an old friend, the contraction mapping principle.

**Example 4**

\[
\begin{align*}
\text{discuss} & \quad \exists! \\
\text{IVP} & \quad \frac{dy}{dx} = x^3 e^{xy} + 7 \\
y(0) &= 0
\end{align*}
\]