Math 2280-1
Tues 1/10
§1.2-1.3

• finish / postpone parts of Monday notes

1^{st} order DE's:

$$F(x, y, \tfrac{dy}{dx}) = 0$$

e.g. $xy + (y')^2 = 0$

can usually use algebra to rewrite as

$$\frac{dy}{dx} = f(x,y)$$

If we also want to specify

$$y(x_0) = y_0$$

Then the DE together with the specified y-value ("initial value") is called an initial value problem:

$$\text{IVP} \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

example 1:

$$\text{IVP} \begin{cases} \frac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$$

Notice: this DE is of the form $y' = f(x)$
(y' only depends on x, not also on $y(x)$)
(such DE's are focus of §1.2)

You can solve this IVP by antidifferentiation:

ans: $y = \frac{x^2}{2} - 3x + \frac{9}{2}$

example 2

$$\text{IVP} \begin{cases} \frac{dy}{dx} = y - x \\ y(0) = 0 \end{cases}$$

magic (actually §1.5) (not §1.2)

$y = x + 1 + Ce^x$ solves DE

check:

solve IVP:

ans: $y = x + 1 - e^x$

slope fields

for any D.E. of the form

(*) $\frac{dy}{dx} = f(x,y)$

geometric interpretation of (*):

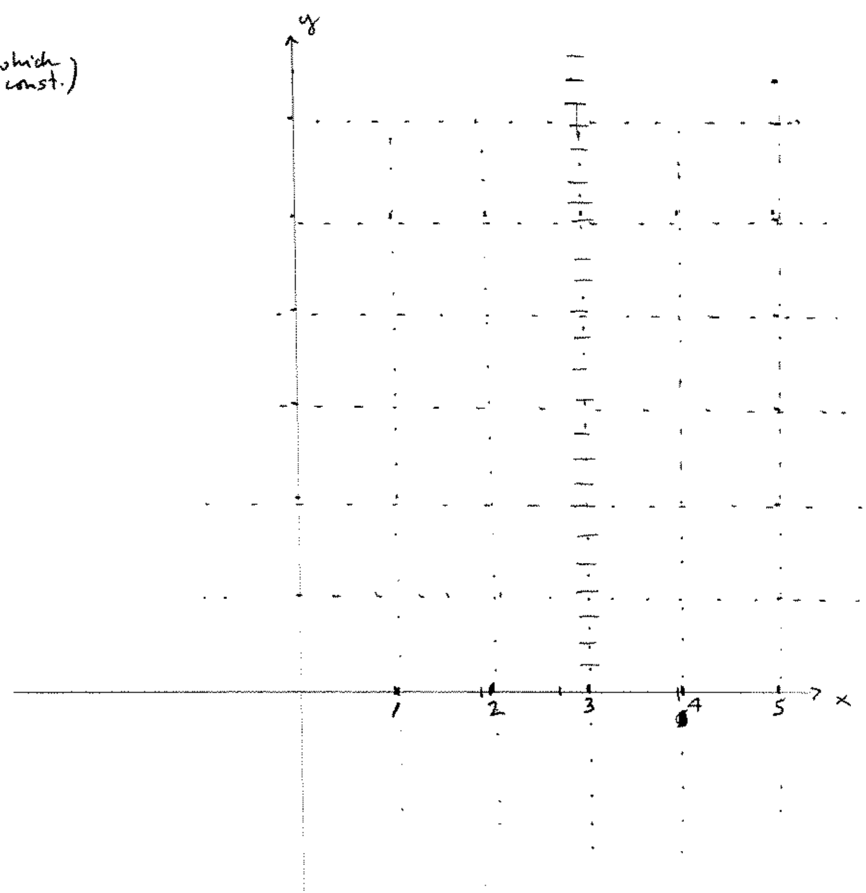
If $y(x)$ solves (*), the slope of its graph $y=y(x)$ at $(x, y(x))$ is determined by the point $(x, y(x))$, i.e. slope $= f(x,y)$

So if you draw a picture of the slope field $m = f(x,y)$ in the x-y plane, solution graphs $y=y(x)$ will be tangent to the field.

example 1 :

- draw the slope field corresponding to the DE $\frac{dy}{dx} = x-3$; i.e. $m(x,y) = x-3$
- sketch the IVP sol'n from page 1 onto this slope field.

slope value m	equation of corresponding isocline (curve on which slope is const.)
0	$x-3=0$ or $x=3$
1	$x=4$
2	$x=5$
-1	$x-3=-1$ or $x=2$
-2	$x=1$.



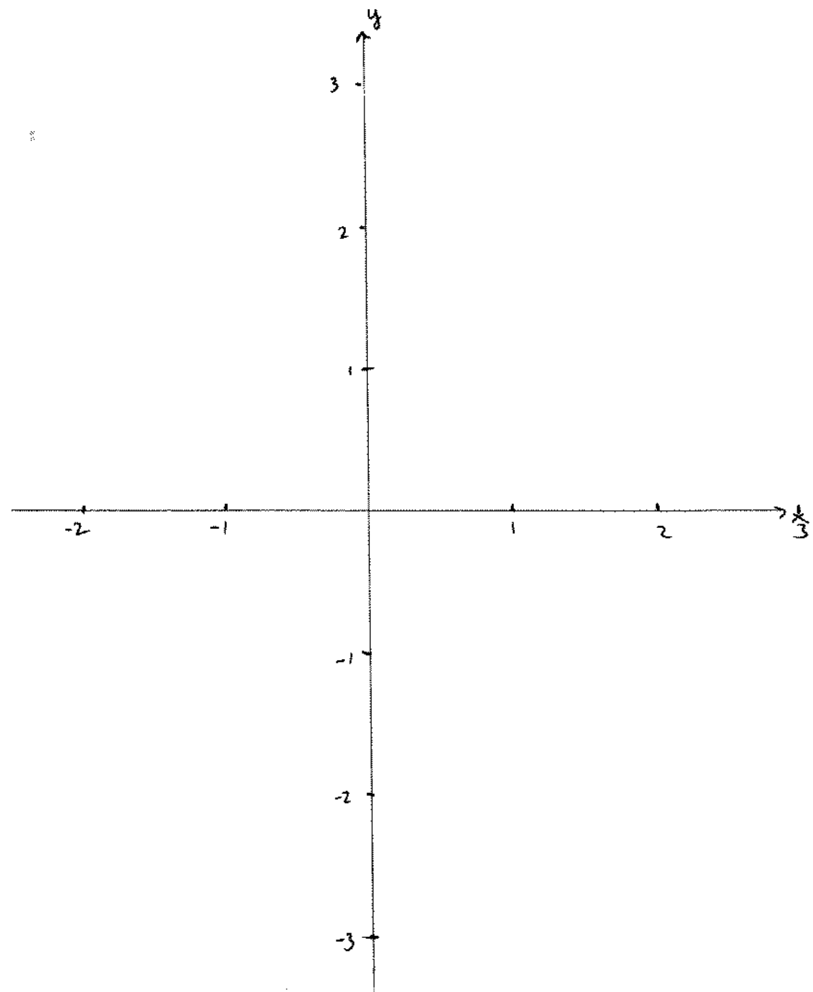
example 2

Make the slope field picture for example 2,

$$\frac{dy}{dx} = y - x$$

Then sketch the IVP solution onto the slope field

slope value	isocline equation
0	
1	
-1	
2	
-2	



Back to §1.2!

- velocity-acceleration problems

  if $x(t)$ is position fcn along an axis, at time t

  $x'(t) := v(t)$ is velocity

  $v'(t) = x''(t) := a(t)$ is acceleration.

If you're given $a(t)$ or $v(t)$ as fcns of t alone, then you antidifferentiate to find $x(t)$.

Example 2 p.13

$x''(t) = a$ (const.)

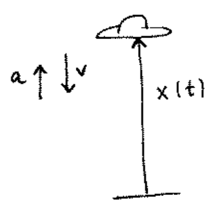
$x'(t) = at + v_0$

$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

lunar lander descending to moon at speed 450 m/s when (additional) retro rockets are fired

they provide a deceleration of 2.5 m/s^2.

At what height should they be fired so that $v=0$ exactly when landing happens?



units

	English	metric
Force	lb	N
mass	slug	kg
distance	ft	m
time	s	s
g	32 ft/sec^2	9.8 m/s^2

g → accel of gravity, on earth

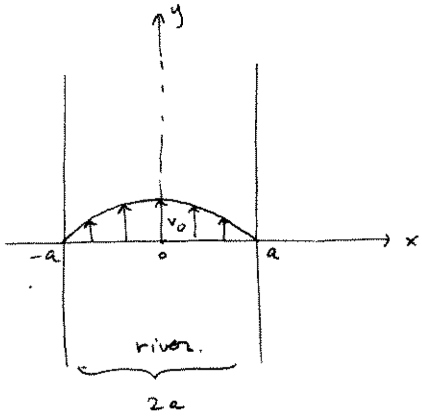
see conversion discussion p. 14.

ans: $t = 180$ sec

$x_0 = 40{,}500$ m

$= 40.5$ km

<u>swimmer problem</u> (Examples 3-4 p.15-16).



$v_R(x) = v_0(1 - x^2/a^2)$ river velocity

swimmer starts at $(-a, 0)$
and swims due East with const veloc v_s
(aims)

where does swimmer land

$$\frac{dx}{dt} = v_s$$

$$\frac{dy}{dt} = v_R$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{v_R}{v_s} = \frac{v_0}{v_s}(1 - x^2/a^2)$$

chain rule!
$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

so, by antidiff,

$$y = \frac{v_0}{v_s}\left[x - \frac{x^3}{3a^2} + C\right]$$

$$y(-a) = 0 = \frac{v_0}{v_s}\left[-a + \frac{a}{3} + C\right] \; ; \; C = \frac{2}{3}a$$

$$\text{so} \quad y(a) = \frac{v_0}{v_s}\left[a - \frac{a}{3} + \frac{2}{3}a\right] = \boxed{\frac{4}{3}\frac{v_0}{v_s}a}$$

<u>example 4</u> river is 1 mile wide
(so a =)

$v_0 = 9$ mi/h
$v_s = 3$ mi/h (excellent!)

where does swimmer land?

ans $y(a) = 2$ mi.