Math 2280-1
Tuesday 4/25

tomorrow is a course review day!

wave eqtn

$$y_{tt} = a^2 y_{xx}$$

a = speed

$a = \sqrt{\frac{T}{\rho}}$ transverse wave on a string

$a = \sqrt{-T'(\rho)}$ parallel (compression) wave.

slinky math theory in yesterday's notes is beautiful, but our slinky failed to be Hooke'sy enough, so I've been hard at work trying to refine the theory:

Slinky computations
April 25 2006

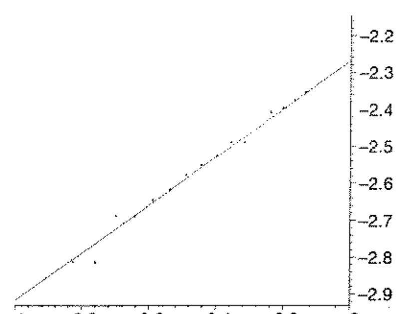
```
> Digits:=5:
  with(plots):
  with(linalg):
> m:=.128; #slinky weighs 128 g
  m0:=m/43; #mass per loop
  g:=9.8;
```

$$m := 0.128$$
$$m0 := 0.0029767$$
$$g := 9.8$$

```
> FSpoints:=[[15*m0*g,.06],[16*m0*g,.06],
  [17*m0*g,.068],[18*m0*g,.068],[19*m0*g,.071],[20*m0*g,.073],[21*m0
  *g,.076],
  [22*m0*g,.078],[23*m0*g,.08],[24*m0*g,.083],[25*m0*g,.083],
  [27*m0*g,.09],[28*m0*g,.091],[29*m0*g,.093],[30*m0*g,.095],[35*m0*
  g,.105],
  [42*m0*g,.115]];
```

FSpoints := [[0.43758, 0.06], [0.46675, 0.06], [0.49592, 0.068], [0.52510, 0.068], [0.55427, 0.071], [0.58344, 0.073], [0.61261, 0.076], [0.64178, 0.078], [0.67096, 0.08], [0.70013, 0.083], [0.72930, 0.083], [0.78764, 0.09], [0.81682, 0.091], [0.84599, 0.093], [0.87516, 0.095], [1.0210, 0.105], [1.2252, 0.115]]

```
> FSmatrix:=convert((FSpoints,matrix)):
> lnlnFS:=map(ln,FSmatrix):
> plot1:=pointplot(lnlnFS):
> xs:=convert(col(lnlnFS,1),list):
  ys:=convert(col(lnlnFS,2),list):
> with(stats):
  fit[leastsquare[[x,y]]]([xs,ys]);
> linefit:=plot(-2.2699+.64683*x,x=-1..0):
  display(plot1,linefit);
```



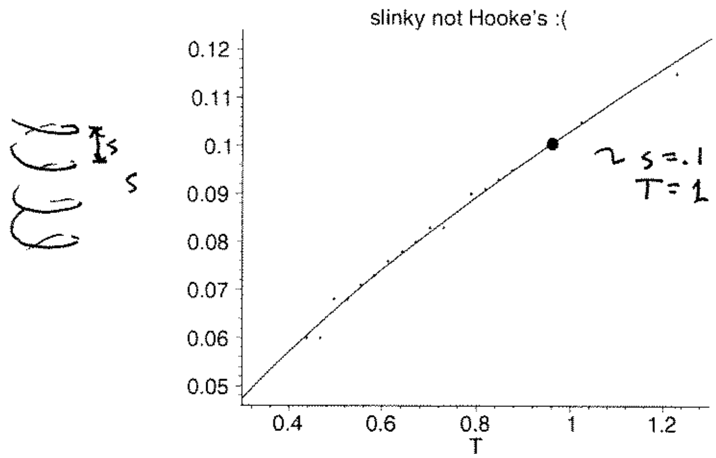
```
> C:=exp(-2.2699):
  p:=.64683;
  stretch:=T->C*T^p;
```

$$C := 0.10332$$

$$p := 0.64683$$

$$stretch := T \to C\,T^p$$

```
> plot2:=pointplot(FSpoints):
  plot3:=plot(stretch(T),T=0.3..1.3,color=black):
  display({plot2,plot3},title='slinky not Hooke's :(');
```




```
> F:=s->(s/C)^(1/p):
> F(s); #tension as a function of stretch
```

$$33.425\, s^{1.5460}$$

```
> F(m0/rho);  #tension as a function of rho
```

$$0.0041539 \left(\frac{1}{\rho}\right)^{1.5460}$$

```
> sqrt(F(.1)/(m0/.1)); #transverse wave speed estimate
     #when stretch is .1 m, using fit function
  2*40*.1/%; #time estimate for one cycle
```

$$5.6515$$

$$1.4155$$

```
> diff(F(m0/rho),rho);
```

$$-\frac{0.0064219 \left(\frac{1}{\rho}\right)^{0.5460}}{\rho^2}$$

```
> subs(rho=m0/.1,%);
```

$$-49.378$$

```
> sqrt(49.378); #parallel speed wave estimate when
     #stretch is .1 m


  (2*40*.1)/sqrt(49.378); #time estimate for one cycle
```

$$7.0269$$

$$1.1385$$

How will our experiment go????

Every sol'n to wave eqtn on real line is a superposition of waves traveling to the right & left with speed a :

if
$$u(x,0) = f(x)$$
$$u_t(x,0) = g(x)$$
$$u_{tt} = a^2 u_{xx} \qquad x \in \mathbb{R},\ t > 0$$

then

$$u(x,t) = \tfrac{1}{2}\left(f(x-at) + f(x+at)\right) + \frac{1}{2a}\left(H(x+at) - H(x-at)\right)$$

$$H(z) := \int_0^z g(s)\,ds$$

$$= \frac{1}{2a}\int_{x-at}^{x+at} g(s)\,ds$$

"d'Alembert solution"

Slinky experiment:

1. measure transverse wave speed two ways:
   with an impulse
   & with the fundamental mode

2. measure compression wave speed.

3. notice that with fixed endpoints reflected waves come back upside down. Explain!

model:
with s = .1
L = 40 (.1) = 4
period
T = 1.42 sec.
speed = 5.65 m/s

model
with s = L
L = 4
period T = 1.14 sec.
speed = 7.03 m/s.