

TEMPLATE FOR YOUR ANSWERS FOR PROJECT 2 -- PART C

In this section of the project you will study the nonlinear, forced, damped oscillator equation for torsional motion

$$x'' + c x' + k \sin(x)\cos(x) = F \cos wt \quad x(0) = a, x'(0) = b$$

This equation cannot be solved analytically, and therefore to study it we need to solve it numerically.

To solve an equation numerically and then to plot its solution, one simple method is to use the numerical capabilities of dsolve. The following example shows how to do this for the equation you worked with in Part A of this project.

define the parameters of the equation

```
with(plots): F := 10; w := 5; k := 25; c:= 7;
```

define the differential equation

```
de := diff(x(t),t,t) + c*diff(x(t),t) + k*x(t) = F*cos(w*t);
```

solve the equation using the initial conditions $x(0)=0$ $dx/dt(0) = 0$

```
p := dsolve( {de,x(0)=0,D(x)(0)=0},x(t),type=numeric):
```

Plot the system from $t=0$ to $t=20$

```
odeplot(p, [t,x(t)], 0..20, numpoints=200, title='oscillations vs. time');
```

```
> restart: with(plots):
```

C.0. Set the parameters in the nonlinear equation equal to the following:
 $c = 0.05$, $k = 2.4$, $w = 1.2$ and $F = 0.06$.

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C.1. Numerically solve the linear equation $x'' + c x' + k x = F \cos wt$ with the same parameters and over the same time period. Plot your solution from $t=150$ to $t=250$.

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[C.2. Numerically solve the nonlinear equation using the initial conditions
 $x(0) = 0$ and $x'(0) = 0$. Plot your solution from $t=150$ to $t=250$.

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[C.3. Numerically solve the nonlinear equation using the initial conditions
 $x(0) = 1.2$ and $x'(0) = 0$. Plot your solution from $t=200$ to $t=300$.

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[C.4. IN DETAIL, describe the differences between the three solutions. What
 can you say about the long time solution of the nonlinear differential
 equation vs. the linear differential equation under similar conditions,
 and about the differences between the two solutions of the nonlinear
 equation? This is the most important part of this exercise, so please
 put some thought into it.

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