

Math 2250  
Maple Project 2 Part A and B

> restart:

### A.1. FREE OSCILLATIONS.

Consider the general problem of free linear oscillations

$$m * x'' + c * x' + k * x = 0$$
$$x(0) = x_0, x'(0) = v_0$$

Using the Maple command "dsolve", find the solution of this initial value problem (it will depend on 5 parameters m, c, k, x0, v0).

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A.1.a. Overdamped oscillations. Suggest an example of the values of parameters m, k, c, so that the oscillations are overdamped.

Using the Maple command "subs" find the solution.

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A.1.b. Find the values of parameters x0, v0, so that the solution increases at first and then decays, staying positive all the time. Plot this solution.

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A.1.c. Find the values of parameters x0, v0, so that the solution changes its sign once and then decays monotonically. Plot this solution.

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### A.2. Underdamped oscillations.

Again consider the general problem of free linear oscillations

$$m * x'' + c * x' + k * x = 0$$
$$x(0) = x_0, x'(0) = v_0$$

A.2.a. Find the values of parameters m, c, k so the solution

changes its sign infinitely many times (decaying to zero).  
Choose values of initial conditions  $x_0, v_0$  and plot the  
corresponding solution.

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A.2.b. Estimate from the graph the pseudoperiod of this solution.

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## PART B: FORCED OSCILLATIONS

> restart:

Consider the general problem of forced linear oscillations

$$m * x'' + c * x' + k * x = F * \cos(w * t)$$
$$x(0)=x_0, x'(0)=v_0$$

B.1. Using the Maple command "dsolve", find the solution of  
this initial value problem (it will depend on 7 parameters  $m$ ,  
 $c$ ,  $k$ ,  $x_0$ ,  $v_0$ ,  $w$ ,  $F$ ).

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B.2. Undamped forced oscillations ( $c=0$ ).

B.2.a. Choose the parameters  $m$ ,  $k$ ,  $w$ , so that the forcing  
frequency  $w$  is 3 times larger than the natural frequency.  
Choose the values of initial conditions and a time-interval which  
allow to see that the oscillations are clearly not harmonic.  
Plot the corresponding solution on this interval.

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B.2.b. Find the period of these oscillations?

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B.2.c. Suggest a set of parameters  $m$ ,  $k$ ,  $F$ ,  $w$ , so the oscillations  
display resonance. Show the resonance on a graph.

The substitution of these resonance parameters into the general  
formula will lead to the Maple diagnostics: Division by zero.  
So you need to solve the differential equation specifically for  
these values of parameters.

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B.3. Damped forced oscillations ( $c > 0$ ).

B.3.a. Choose a set of the values of parameters  $m$ ,  $c > 0$ ,  $k$ ,  $F$ ,  $w$ ,  $x_0$ ,  $v_0$  and plot the corresponding solution.

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B.3.b. What is the long time solution? Plot it

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B.3.c For general  $m$ ,  $c$ ,  $k$ ,  $f$ ,  $w$ , determine the amplitude  $A$  of long time oscillations. (Hint: solve the differential equation zero initial condition).

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B.3.d Choose  $F = 10$ ,  $m = 1$ ,  $k = 25$ , and consider several values of  $c$ , say  $c = 2$ ,  $c = 1$ ,  $c = 1/2$ . On the same graph plot the corresponding amplitudes  $A(w)$ ,  $w = 0..30$ . Use different colors for different curves. You will probably be unable to print a color picture, but you can see it on the screen. (You will need to use command "display" in order to show these pictures on the same graph. You can use "help" to find out about this command).

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B.3.e. You should see that the maximum amplitude becomes larger and larger when  $c \rightarrow 0$ . Determine whether the frequency  $w$  where the amplitude reaches its maximum is greater or less than the natural frequency.

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