

masses, springs, washboarding cars.  
Math 2280-2  
March 6, 2001

**Part I:** two equal masses suspended with three identical springs:

```
> restart:with(linalg):  
Warning, the protected names norm and trace have been redefined and unprotected  
> A:=matrix(2,2,[-2*k/m, k/m,k/m,-2*k/m]);  
#this should be the "A" matrix you get for  
#our two-mass, three-spring system.  

$$A := \begin{bmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{bmatrix}$$
  
> eigenvects(A);  

$$\left[ -3\frac{k}{m}, 1, \{[-1, 1]\} \right], \left[ -\frac{k}{m}, 1, \{[1, 1]\} \right]$$

```

Here is the data from our experiment..... we'll recheck the timing data in class.

- (1) The "in-phase" characteristic motion oscillated at 50 cycles per 30 seconds.
- (2) The "out of phase" oscillation was still 100 cycles in 33 seconds.
- (3) The mass of each ball 23 grams. (I checked this with Ziggy.)
- (4) The mass of each spring is 9 grams
- (4) A mass of 20 grams stretches a single spring by 5.9 cm.

Notice, our theory predicts natural angular frequencies of  $\sqrt{3k/m}$  and  $\sqrt{k/m}$ , so the ratio of our experimental frequencies should be  $\sqrt{3}$ :

```
> f1:=100/33.0;  
f2:=50/30.0;  
f1/f2;  
sqrt(3.0);  
 $f1 := 3.030303030$   
 $f2 := 1.666666667$   
1.818181818  
1.732050808
```

**HOMEWORK:** (We did this in class, so don't bother handing it in.) Ignore the mass of the springs. Compute the theoretical natural frequencies (in cycles/sec). **Answer:** You will get 1.91 and 3.31 as your predictions. These numbers are close, but are 9% and 15% too large, respectively. Here's the details:

```
> m:=.023;  
k:=.02*9.8/.059;  
 $m := .023$   
 $k := 3.322033898$ 
```

```

[ > evalf(sqrt(k/m)/(2*Pi));
  evalf(sqrt(3*k/m)/(2*Pi));
                                     1.912750143
                                     3.312980432

```

**EXPLANATION:** The springs actually have mass, equal to 9 grams each, which you did not take into account. This is on the same order of magnitude as the ball masses, and causes the actual experiment to run more slowly than our model predicts. In order to be more accurate the total energy of our model must account for the kinetic energy of the springs. You actually have the tools to model this more-complicated situation, using the ideas of total energy discussed in section 5.6, and a little Calculus. I carried out such an analysis, assuming that the spring velocity at a point on the spring linearly interpolates the velocity of the wall and mass (or mass and mass) which bounds it. Based on this analysis, the improved model predicts the two fundamental frequencies to be 1.68 cycles per second and 2.99 cycles per second. This agrees with our experimental results to within 1%, which is pretty amazing. **I offer a prestigious Math Department T-shirt to the first student or group of students to submit this "corrected" model and results, including its derivation.**

[ Part 2: Washboarding revisited. (The two-axle automobile page 332)

```

[ > restart:with(linalg):
  Warning, the protected names norm and trace have been redefined and unprotected

```

[ Here is the matrix A which we deduce from equation 40 on page 332.

```

[ > A:=matrix(2,2,[-(k1+k2)/m, (k1*L1 - k2*L2)/m,
  (k1*L1-k2*L2)/In, -(k1*L1^2+k2*L2^2)/In]);

```

$$A := \begin{bmatrix} -\frac{k1+k2}{m} & \frac{k1 L1 - k2 L2}{m} \\ \frac{k1 L1 - k2 L2}{In} & -\frac{k1 L1^2 + k2 L2^2}{In} \end{bmatrix}$$

[ Let's do the system which results from the data in #24):

```

[ > m:=100;
  In:=1000;
  L1:=6;
  L2:=4;
  k1:=2000;
  k2:=2000;
                                     m := 100
                                     In := 1000
                                     L1 := 6
                                     L2 := 4
                                     k1 := 2000
                                     k2 := 2000

```

```

[ > eigenvects(A);
  [-72 + 4*sqrt(74), 1, {[1, -4/5 + 1/10*sqrt(74)]}], [-72 - 4*sqrt(74), 1, {[1, -4/5 - 1/10*sqrt(74)]}]

```

```

> evalf(%);
      [-37.59069893, 1., {[1., .0602325267]}], [-106.4093011, 1., {[1., -1.660232527]}]
> pi:=evalf(Pi);
       $\pi := 3.141592654$ 
> f1:=sqrt(37.59069893)/(2*pi);
      f2:=sqrt(106.4093011)/(2*pi);
       $f1 := .9757989175$ 
       $f2 := 1.641760970$ 
> fforce:=v/L;
      #forcing requency is speed divided
      #by wavelength of washboard (more like freeway
      #syndrome than off-road case here...
       $fforce := \frac{v}{L}$ 
> L:=40;
      solve(f1=v/L,v);
      solve(f2=v/L,v);
      #critical car speeds in feet/sec
       $L := 40$ 
      39.03195670
      65.67043880
> 39.03195670*(1/5280)*3600;
      65.67043880*(1/5280)*3600;
      #speeds in miles/hour
      26.61269775
      44.77529920

```