Math 2280-2
Tuesday March 27
Linearization and stability

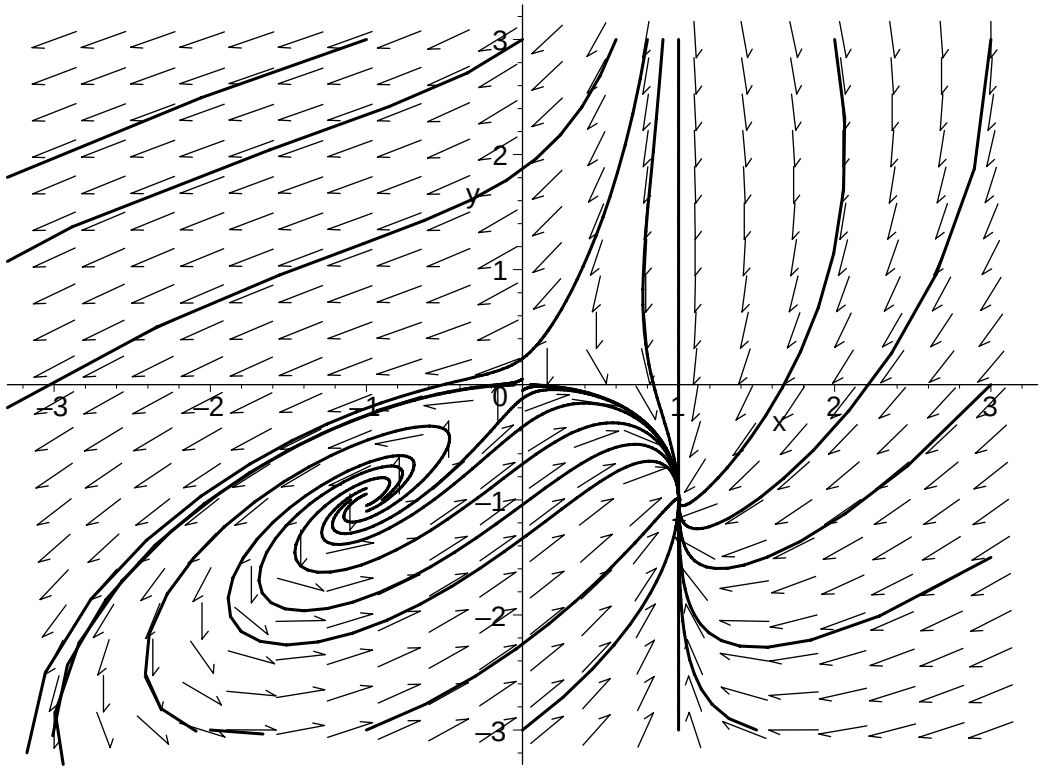
We work with your homework problem #8, page 384

```
> restart:
> with(plots):with(linalg):with(DEtools):
> deqtn:={diff(x(t),t)=x(t)-y(t)-x(t)^2+x(t)*y(t),
          diff(y(t),t)=-y(t)-x(t)^2};
```

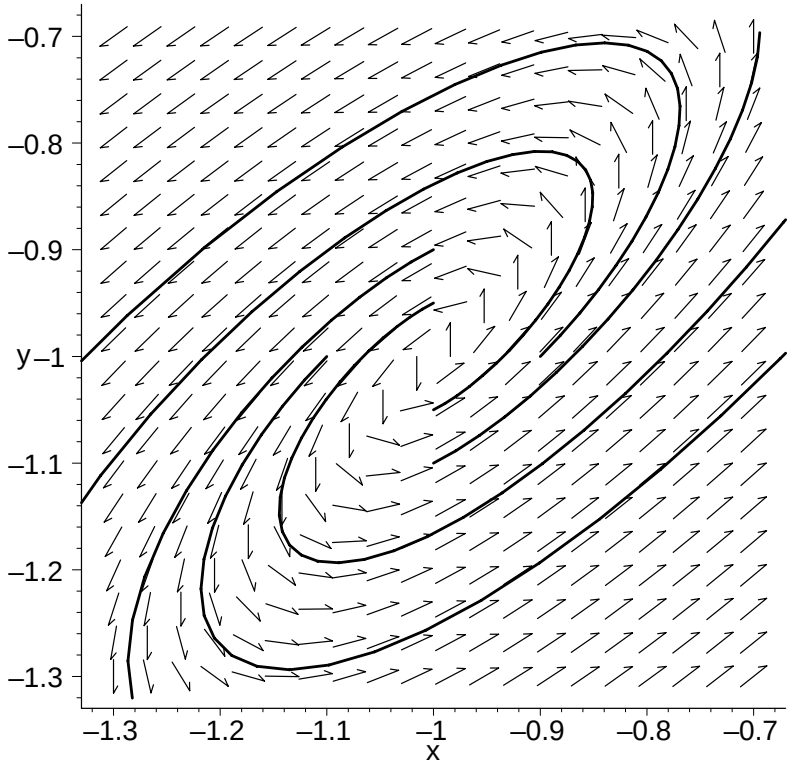
$$deqtn := \{\frac{\partial}{\partial t}\,y(t) = -y(t) - x(t)^2,\ \frac{\partial}{\partial t}\,x(t) = x(t) - y(t) - x(t)^2 + x(t)\,y(t)\}$$

```
> ICs:=[[x(0)=-1,y(0)=3],[x(0)=0,y(0)=3],
  [x(0)=.9,y(0)=3],
  [x(0)=.6,y(0)=3],[x(0)=.8,y(0)=3],
  [x(0)=1,y(0)=3],[x(0)=2,y(0)=3],
  [x(0)=3,y(0)=3],[x(0)=3,y(0)=0],
  [x(0)=3,y(0)=-1.5],[x(0)=0.05,y(0)=0],
  [x(0)=-.05,y(0)=0],[x(0)=0,y(0)=-.05],
  [x(0)=0,y(0)=.05],
  [x(0)=1.5,y(0)=-3],
  [x(0)=1,y(0)=-3],[x(0)=0,y(0)=-3],
  [x(0)=-1,y(0)=-3],[x(0)=-2,y(0)=-3],
  [x(0)=-1.1,y(0)=-1],[x(0)=-1,y(0)=-.9],
  [x(0)=-.9,y(0)=-1],[x(0)=-1,y(0)=-1.1],
  [x(0)=-1,y(0)=-.95],[x(0)=-1,y(0)=-1.05]]:
  #initial conditions to make a picture like Figure 6.1.16
> DEplot(deqtn,[x(t),y(t)],t=0..15,ICs,x=-3..3,y=-3..3,arrows=small,

  stepsize=.1,color=black,linecolor=black);
  #Figure 6.1.16
```



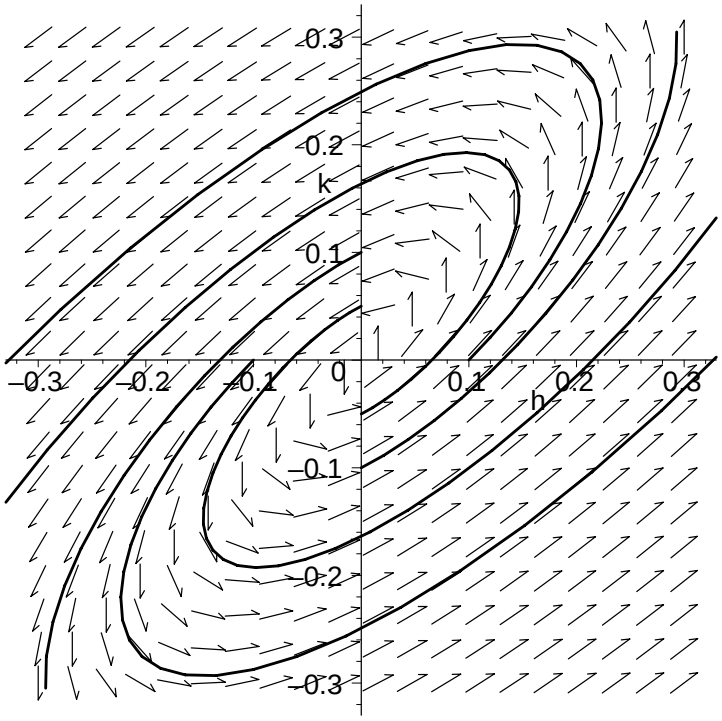

```
> 
> 
> ICs1:=[
  [x(0)=-1.1,y(0)=-1],[x(0)=-1,y(0)=-.9],
  [x(0)=-.9,y(0)=-1],[x(0)=-1,y(0)=-1.1],
  [x(0)=-1,y(0)=-.95],[x(0)=-1,y(0)=-1.05]]:
  #look near the equilibrium point [-1,-1]
> DEplot(deqtn,[x(t),y(t)],t=0..5,ICs1,x=-1.3..-.7,y=-1.3..-.7,arrow
  s=small,
  stepsize=.1,color=black,linecolor=black);
  #solution to nonlinear system near [-1,-1]
```




```
> deqtn2:={diff(h(t),t)=2*h(t)-2*k(t),diff(k(t),t)=2*h(t)-k(t)};
  #the linearization near [-1,-1]
```

$$deqtn2 := \{\frac{\partial}{\partial t}\,\mathrm{h}(t) = 2\,\mathrm{h}(t) - 2\,\mathrm{k}(t), \frac{\partial}{\partial t}\,\mathrm{k}(t) = 2\,\mathrm{h}(t) - \mathrm{k}(t)\}$$

```
> ICs2:=[
  [h(0)=-.1,k(0)=0],[h(0)=0,k(0)=.1],
  [h(0)=.1,k(0)=0],[h(0)=0,k(0)=-.1],
  [h(0)=0,k(0)=.05],[h(0)=0,k(0)=-.05]]:
  #initial conditions for [h,k]
> DEplot(deqtn2,[h(t),k(t)],t=0..5,ICs2,h=-.3..(.3),k=-.3..(.3),arro
  ws=small,
  stepsize=.1,color=black,linecolor=black);
  #solution to linearized equation, should look like
  #nonlinearized one
```


0.3
0.2
k
0.1
–0.3
–0.2
–0.1
0
0.1
h
0.2
0.3
–0.1
–0.2
–0.3