

Math 2280-2
 Friday 23 March, 2001

Inhomogeneous linear systems of differential equations

Work for example 2 page 368

```
[ > with(linalg):
> A:=matrix(3,3,[-.5,0,0,.5,-.25,0,0,.25,-.2]);
      A :=  $\begin{bmatrix} -.5 & 0 & 0 \\ .5 & -.25 & 0 \\ 0 & .25 & -.2 \end{bmatrix}$ 
> eigenvects(A);
      [-.5, 1, {[1, -2.000000000, 1.666666667]}], [-.25, 1, {[0, 1, -5.000000000]}], [-.2, 1, {[0, 0, 1]}]
> k:=evalm(inverse(A)*vector([-20,0,0]));
      k := [40.00000000, 80.00000000, 100.0000000]
> Phi0:=matrix(3,3,[3,0,0,-6,1,0,5,-5,1]);
      Phi0 :=  $\begin{bmatrix} 3 & 0 & 0 \\ -6 & 1 & 0 \\ 5 & -5 & 1 \end{bmatrix}$ 
> c:=evalm(-inverse(Phi0)*k);
      c := [-13.33333333, -160.0000000, -833.3333333]
```

Work for example 4 page 372

```
[ > A:=matrix(2,2,[4,2,3,-1]);
      A :=  $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$ 
> x0:=vector([7,3]);
      x0 := [7, 3]
> f:=t->evalm(-t*exp(-2*t)*vector([15,4]));
      #the inhomogeneous term.
      f := t -> evalm(-t e(-2t) [15, 4])
> sol:=t->evalm(exponential(A,t)*&*(x0+
int(exponential(-A,s)*&f(s),s=0..t)));
      #solution formula in Maple
      sol := t -> evalm( '&*' ( exponential(A, t), x0 +  $\int_0^t$  '&*' ( exponential(-A, s), f(s) ) ds ) )
> sol(t);
```

$$\left[\left(\frac{1}{7} e^{(-2t)} + \frac{6}{7} e^{(5t)} \right) \left(7 - \frac{1}{14} \frac{-28t - 4 + 7t^2 (e^t)^7 + 4(e^t)^7}{(e^t)^7} \right) \right. \\
+ \left(\frac{2}{7} e^{(5t)} - \frac{2}{7} e^{(-2t)} \right) \left(3 + \frac{1}{14} \frac{(21t^2 (e^t)^7 + 14t + 2 - 2(e^t)^7)}{(e^t)^7} \right) \\
\left. \left(\frac{3}{7} e^{(5t)} - \frac{3}{7} e^{(-2t)} \right) \left(7 - \frac{1}{14} \frac{-28t - 4 + 7t^2 (e^t)^7 + 4(e^t)^7}{(e^t)^7} \right) \right. \\
\left. + \left(\frac{6}{7} e^{(-2t)} + \frac{1}{7} e^{(5t)} \right) \left(3 + \frac{1}{14} \frac{(21t^2 (e^t)^7 + 14t + 2 - 2(e^t)^7)}{(e^t)^7} \right) \right] \\
> \text{simplify(\%)}; \\
\# \text{this is the solution at the bottom of page 372} \\
\left[\frac{3}{7} e^{(-2t)} - \frac{1}{2} e^{(-2t)} t^2 + \frac{46}{7} e^{(5t)} + 2 e^{(-2t)} t, \frac{23}{7} e^{(5t)} + e^{(-2t)} t - \frac{2}{7} e^{(-2t)} + \frac{3}{2} e^{(-2t)} t^2 \right] \\
[>$$