Math 2280-2

Tuesday Jan 9

The geometric interpretation of a first order differential equation is connected to slope fields. Consider the differential equation

$$\frac{dy}{dx} = f(x, y)$$

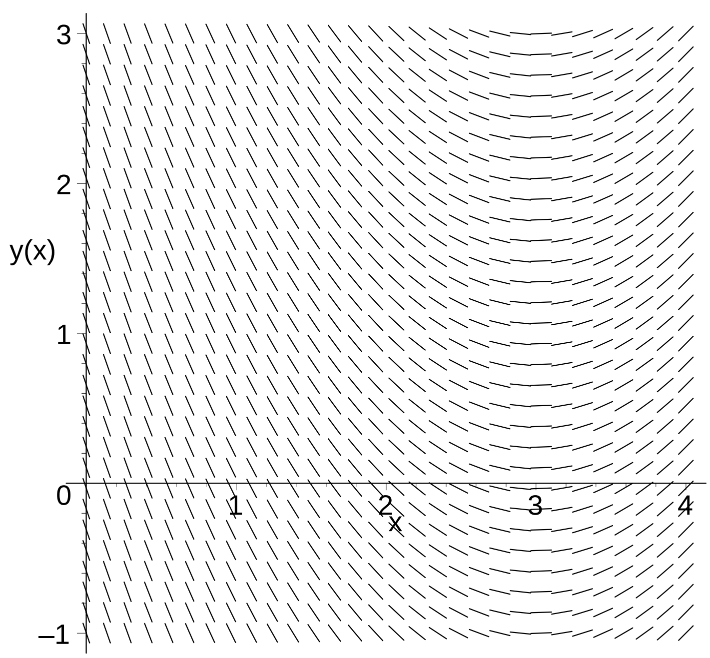
The associated slope field in the x-y plane is a field of slopes, where the slope at point (x,y) is given by the formula f(x,y). A solution y(x) to the differential equation above will have a graph y=y(x), the tangent line slope dy/dx will exactly ewqual f(x,y). This means that we can use the slope field to draw the graph of y(x), even if we don't have a formula for y.

Example 1:

```
> with(DEtools):
     #Maple tools for differential equations
  deqtn:=diff(y(x),x)=x-3;
     #Example 1
```

$$deqtn := \frac{\partial}{\partial x} y(x) = x - 3$$

```
> dfieldplot(deqtn, y(x),x=0..4,y=-1..3, arrows=line,
  color=black, dirgrid=[30,30]);
```

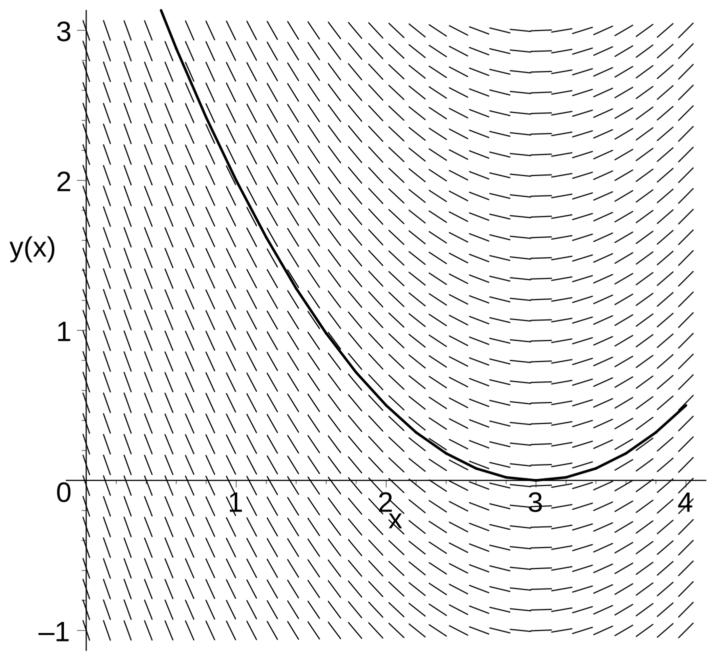


We can plot the solution to the initial value problem y(1)=2 onto our direction field to verify our classwork:

```
> dsolve({deqtn,y(1)=2},y(x));
    #we did this in class by magic
```

$$y(x) = \frac{1}{2}x^2 - 3x + \frac{9}{2}$$

```
> DEplot(deqtn,y(x),x=0..4,{[y(1)=2]},y=-1..3,
      color=black, linecolor=black,
      arrows=line, dirgrid=[30,30]);
```



Second Example:

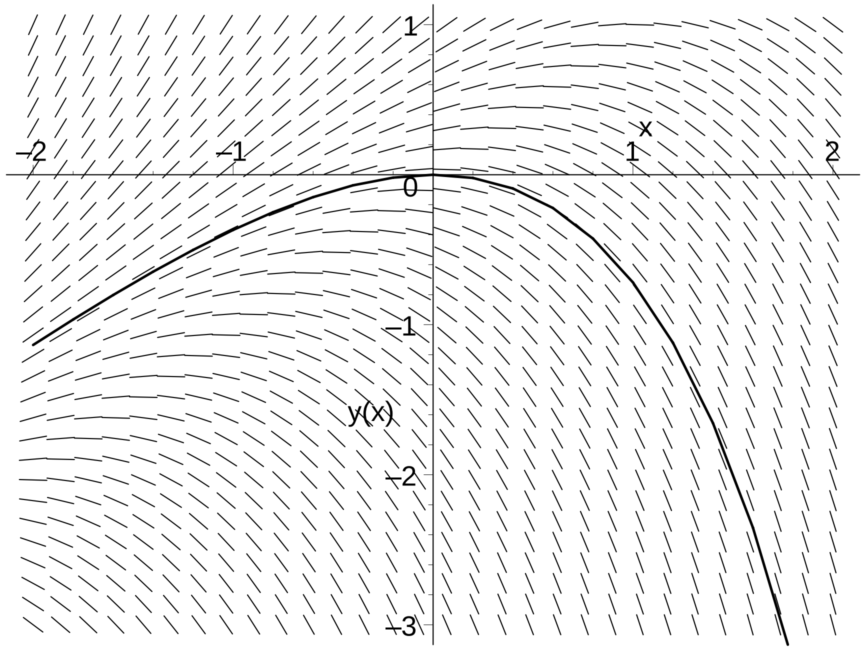
```
> deqtn:=diff(y(x),x)=y(x)-x;
```

$$deqtn := \frac{\partial}{\partial x} y(x) = y(x) - x$$

```
> dsolve({deqtn,y(0)=0},y(x));
```

$$y(x) = x + 1 - \mathbf{e}^x$$

```
> DEplot(deqtn,y(x),x=-2..2,{[y(0)=0]},y=-3..1,
    color=black, linecolor=black,
    arrows=line, dirgrid=[30,30]);
```



1
–2
–1
x
1
2
0
–1
y(x)
–2
–3