Math 2280-2

Tuesday April 10, 2001

Even and Odd extensions

Differentiating and integrating Fourier Series

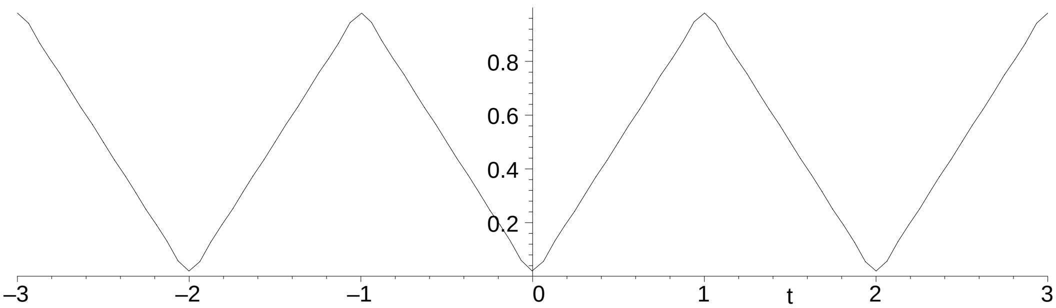
We play with example 1 page 613, taking L=1.

```
> restart:
  with(plots):
```

```
Warning, the name changecoords has been redefined
```

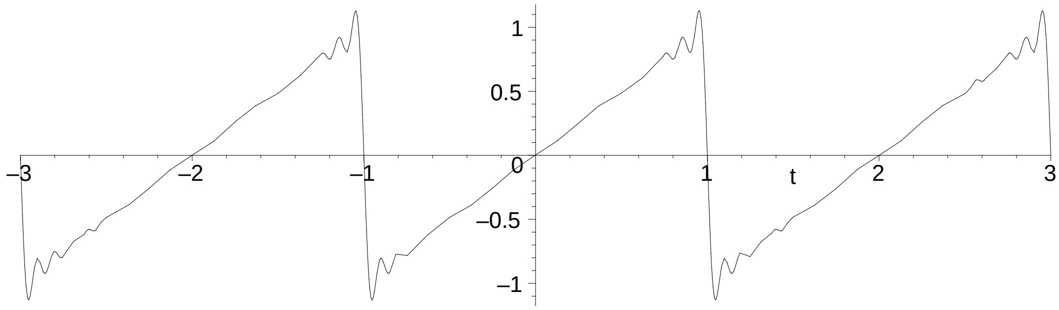
```
> fe:=t->1/2 - (4/Pi^2)*sum(cos(Pi*(2*n-1)*t)/(2*n-1)^2,n=1..5):
  #even extension, truncated
> fe(t);
  plot(fe(t),t=-3..3,color=black);
```

$$\frac{1}{2}-\frac{4\left(\cos(\pi\, t)+\frac{1}{9}\cos(3\,\pi\, t)+\frac{1}{25}\cos(5\,\pi\, t)+\frac{1}{49}\cos(7\,\pi\, t)+\frac{1}{81}\cos(9\,\pi\, t)\right)}{\pi^2}$$




```
> fo:=t->(2/Pi)*sum((-1)^(n+1)*sin(Pi*n*t)/n,n=1..20):
  fo(t);
  plot(fo(t),t=-3..3,color=black);
```

$$2\left(\sin(\pi\, t)-\frac{1}{2}\sin(2\,\pi\, t)+\frac{1}{3}\sin(3\,\pi\, t)-\frac{1}{4}\sin(4\,\pi\, t)+\frac{1}{5}\sin(5\,\pi\, t)-\frac{1}{6}\sin(6\,\pi\, t)+\frac{1}{7}\sin(7\,\pi\, t)\right.$$
$$-\frac{1}{8}\sin(8\,\pi\, t)+\frac{1}{9}\sin(9\,\pi\, t)-\frac{1}{10}\sin(10\,\pi\, t)+\frac{1}{11}\sin(11\,\pi\, t)-\frac{1}{12}\sin(12\,\pi\, t)+\frac{1}{13}\sin(13\,\pi\, t)$$
$$-\frac{1}{14}\sin(14\,\pi\, t)+\frac{1}{15}\sin(15\,\pi\, t)-\frac{1}{16}\sin(16\,\pi\, t)+\frac{1}{17}\sin(17\,\pi\, t)-\frac{1}{18}\sin(18\,\pi\, t)+\frac{1}{19}\sin(19\,\pi\, t)$$
$$\left.-\frac{1}{20}\sin(20\,\pi\, t)\right)/\,\pi$$



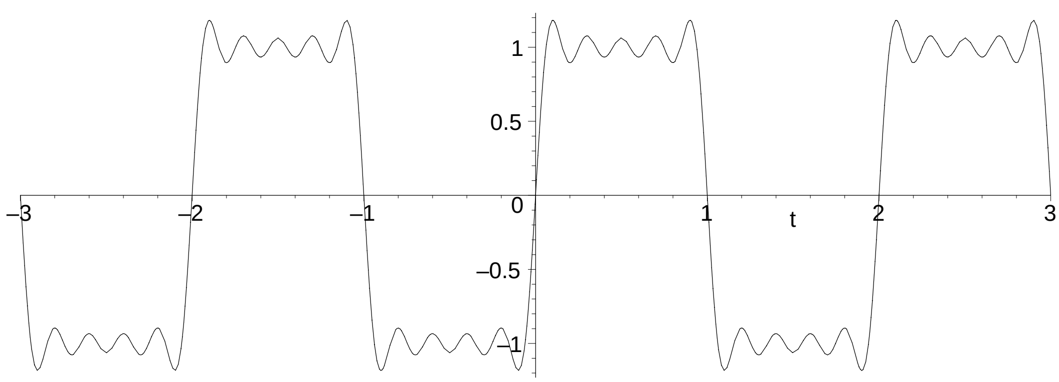
>

What happens when you differentiate or integrate a Fourier series term by term???? Answer: integration is good, differentiation might be bad

```
> fep:=t->diff(fe(t),t);
  #the derivative
  fep(t);
  plot(fep(t),t=-3..3,color=black);
  #this might look familiar, e.g. why should it look
  #like a square wave?
```

$$fep := t \to \mathrm{diff}(\mathrm{fe}(t), t)$$

$$-4\,\frac{-\sin(\pi\,t)\,\pi - \frac{1}{3}\sin(3\,\pi\,t)\,\pi - \frac{1}{5}\sin(5\,\pi\,t)\,\pi - \frac{1}{7}\sin(7\,\pi\,t)\,\pi - \frac{1}{9}\sin(9\,\pi\,t)\,\pi}{\pi^2}$$

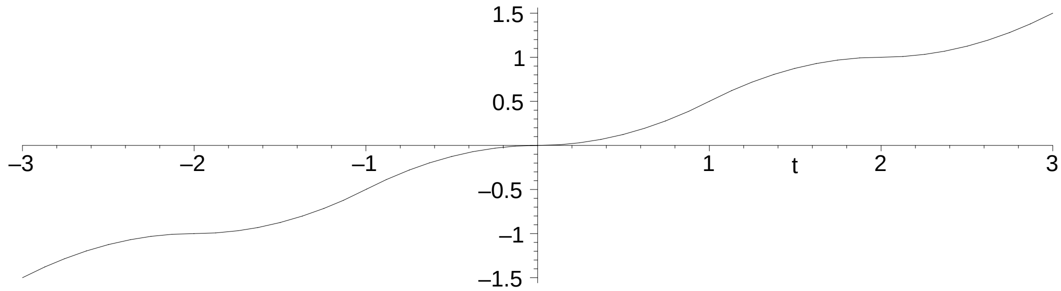



```
> feint:=t->int(fe(s),s=0..t);
  #the integral with F(0)=0
  feint(t);
  #you would get a nicer formula by hand
  plot(feint(t),t=-3..3,color=black);
```

$$feint := t \to \int_0^t \mathrm{fe}(s)\,ds$$

$$-\frac{1}{62511750}\,(-31255875\,t\,\pi^3 + 250047000\sin(\pi\,t) + 9261000\sin(3\,\pi\,t) + 2000376\sin(5\,\pi\,t)$$

$+ 729000 \sin(7 \pi t) + 343000 \sin(9 \pi t)) \Big/ \pi^3$

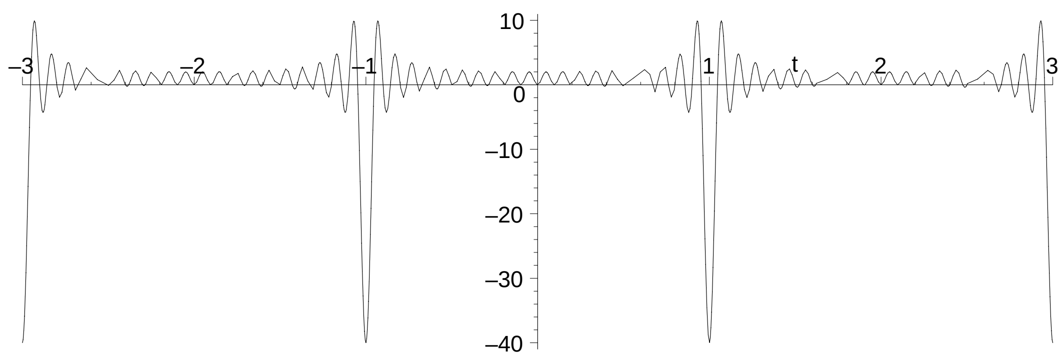


Now the same procedure with the odd extension:

```
> fop:=t->diff(fo(t),t);
  fop(t);
  plot(fop(t),t=-3..3,color=black);
  #this should be interesting
```

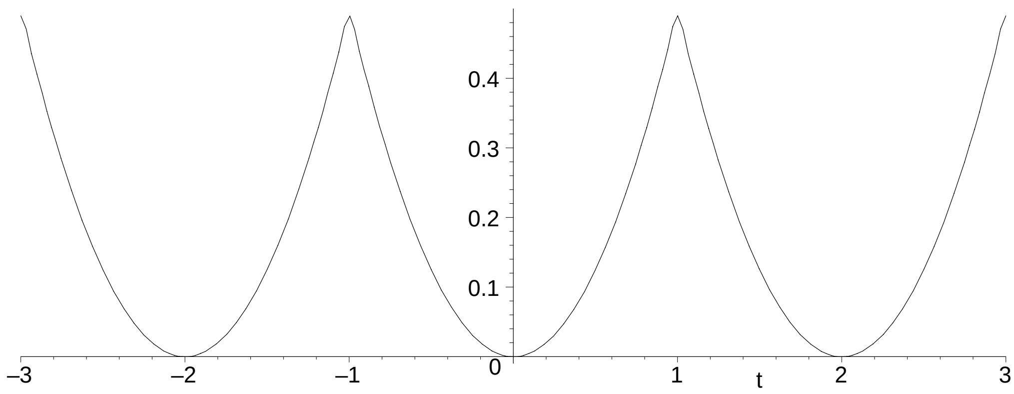
$$fop := t \to \mathrm{diff}(\mathrm{fo}(t), t)$$

$$2\,(\cos(\pi t)\,\pi - \cos(2\pi t)\,\pi + \cos(3\pi t)\,\pi - \cos(4\pi t)\,\pi + \cos(5\pi t)\,\pi - \cos(6\pi t)\,\pi + \cos(7\pi t)\,\pi - \cos(8\pi t)\,\pi + \cos(9\pi t)\,\pi - \cos(10\pi t)\,\pi + \cos(11\pi t)\,\pi - \cos(12\pi t)\,\pi + \cos(13\pi t)\,\pi - \cos(14\pi t)\,\pi + \cos(15\pi t)\,\pi - \cos(16\pi t)\,\pi + \cos(17\pi t)\,\pi - \cos(18\pi t)\,\pi + \cos(19\pi t)\,\pi - \cos(20\pi t)\,\pi) / \pi$$




```
> foint(t):=int(fo(s),s=0..t);
  plot(foint(t),t=-3..3,color=black);
```

$$\mathrm{foint}(t) := \frac{1}{27096187995676800}\,(-54192375991353600 \cos(\pi t) + 13548093997838400 \cos(2\pi t) - 6021375110150400 \cos(3\pi t) + 3387023499459600 \cos(4\pi t) - 2167695039654144 \cos(5\pi t) + 1505343777537600 \cos(6\pi t) - 1105966856966400 \cos(7\pi t) + 846755874864900 \cos(8\pi t) - 669041678905600 \cos(9\pi t) + 541923759913536 \cos(10\pi t) - 447870875961600 \cos(11\pi t) + 376335944384400 \cos(12\pi t) - 320664946694400 \cos(13\pi t) + 276491714241600 \cos(14\pi t) - 240855004406016 \cos(15\pi t) + 211688968716225 \cos(16\pi t) - 187516871942400 \cos(17\pi t) + 167260419726400 \cos(18\pi t) - 150117385017600 \cos(19\pi t) + 135480939978384 \cos(20\pi t) + 44507080864391115) / \pi^2$$


0.4
0.3
0.2
0.1
0
–3
–2
–1
1
t
2
3