Math 2280-1
Tuesday Sept 30

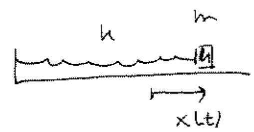
HW: only need to do §3.4 for this week
exam Friday is thru §3.4.

Playday!

(We'll decide whether to finish Monday's notes first or second.)

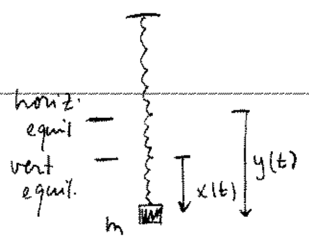
horizontal vs. vertical spring

horizontal, no friction



$$mx'' = -kx$$
$$mx'' + kx = 0$$

now hang the same spring-mass assembly!



By Newton, gravity force.

$$my'' = -ky + mg$$
$$my'' + ky = mg$$
$$y'' + \frac{k}{m}y = g$$

$$y_P = \frac{mg}{k} \quad (\text{is equil sol'n!})$$

so $y = \frac{mg}{k} + C\cos(\omega_0 t - \alpha)$ $\qquad \omega_0 = \sqrt{\frac{k}{m}}$
$(= y_P + y_H)$

or notice that
vert equil when $ky = mg$
$y = \frac{mg}{k}$

so $x = y - \frac{mg}{k}$

and $x'' + \frac{k}{m}x = y'' + \frac{k}{m}\left(y - \frac{mg}{k}\right) = y'' + \frac{k}{m}y - g = 0$

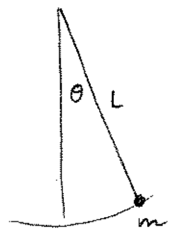
so $\boxed{x'' + \frac{k}{m}x = 0}$ $\qquad \boxed{x(t) = C\cos(\omega_0 t - \alpha)}$

so $x(t)$ satisfies the same eqtn as flat spring-mass displacement fcn, with same Hooke's const.

most important $\rightarrow$ (of course, since $x(t)$ is displacement from (vertical) equil., we could've just used linearization argument to derive this DE, but would need to think about why we get same Hooke's const.).

Experiments

pendulum



$$L\theta'' + g\theta \equiv 0$$

$$\theta'' + \frac{g}{L}\theta = 0$$

$$\omega_0 = \sqrt{\frac{g}{L}} \qquad \theta(t) = C\cos(\omega_0 t - \alpha)$$

$$\nu = \frac{\omega_0}{2\pi}$$

$$T = \frac{2\pi}{\omega_0}$$

prediction :

Experiment notes
Math 2280-1
Tuesday September 30

Pendulum:

```
> restart:
> Digits:=4:
```

Here is my measurement of L, which we'll check again in class. It assumes the effective length of the pendulum uses the the distance to the ball's center of mass, from the top. (And, where exactly is the top?!)

```
> L:=1.532;
  g:=9.806;
  omega:=sqrt(g/L); #radians per second
  f:=evalf(omega/(2*Pi)); #cycles per second
  T:=1/f; #seconds per cycle
```

$$L := 1.532$$

$$g := 9.806$$

$$\omega := 2.529976935$$

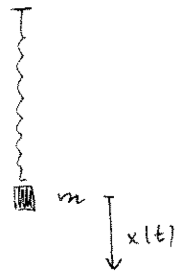
$$f := 0.4026583352$$

$$T := 2.483495094$$

experiment :

possible errors

Mass-spring experiment:



prediction:

Mass-spring:
How Hookes constant:

```
> 88.5-45.5; #add 50g and measure displacement
```

$$43.0$$

```
> m:=.1;  #the mass is 100g = 0.1 kg
```

$$m := 0.1$$

My measurement for k, using a 50g mass and measuring the displacement in meters

```
> k*.43=.05*g;
```

$$0.43\,k = 0.49030$$

```
> k:=.05*g/.43;
```

$$k := 1.140232558$$

```
> omega:=sqrt(k/m);#radians per second
  f:=omega/evalf(2*Pi); #cycles per second
  T:=1/f; #seconds per cycle
```

$$\omega := 3.376732974$$

$$f := 0.5374237442$$

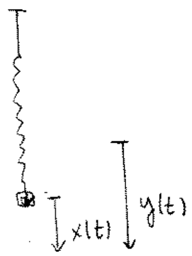
$$T := 1.860729100$$

```
>
```

experiment:

error sources?
(there's one very big one!)

<u>Improved spring</u>: account for mass of spring – it contributes KE if it's moving!



Using $y(t)$,

$$KE + PE = \tfrac{1}{2} m (y'(t))^2 + KE_{spring} + \underbrace{\tfrac{1}{2} k y^2}_{PE_{spring}} \underbrace{- mgy}_{PE_{gravity}} \equiv const$$

(actually another linear term in y for the PE of spring due to gravity)

model:
the top of spring isn't moving. the bottom is moving with ~~speed~~ vel $y'(t)$
assume the ~~speed~~ vel of a piece of spring a proportion μ $(0<\mu<1)$ of the way from top to bottom is $\mu y'(t)$
then the $(speed)^2 = \mu^2 (y'(t))^2$

$$KE = \left[\int_0^1 \mu^2 \, \underbrace{(m_s d\mu)}_{dm}\right] \tfrac{1}{2} y'(t)^2 = \tfrac{1}{6} m_s \, y'(t)^2$$

so, $M := \left(m + \tfrac{1}{3} m_s\right)$

$\Rightarrow KE = \tfrac{1}{2} M \, y'(t)^2$

$\Rightarrow \tfrac{1}{2} M (y')^2 + \tfrac{1}{2} k y^2 + Cy \equiv const$

$\Rightarrow M y' y'' + k y y' + C y' \equiv 0$

$y' \left[M y'' + k y + C \right] \equiv 0$

$M y'' + k y = -C$

$\omega_0 = \sqrt{\frac{k}{M}} \Leftarrow$ "effective mass" $M = m + \tfrac{1}{3} m_s$

<u>new estimate</u>,

```
> ms:=.011: #spring has mass 11g
  M:=m+ms/3;
```

$$M := 0.1036666667$$

```
> omega1:=sqrt(k/M); #new angular freq est.
  f1:=omega1/evalf(2*Pi); #new freq est.
  T1:=1/f1;#new period est
```

$$\omega 1 := 3.316478236$$

$$f1 := 0.5278339048$$

$$T1 := 1.894535366$$