Math 2280-1
Friday Sept 26

HW for Monday Oct 6
Exam 1 is Fri. Oct 3!

3.4 4, 5, 6, 13, 15, 19, 23 32, 33
3.5 3 4, 17, 19 36, 37, 43, 49 50, 51, 64

We will discuss the case of complex roots to the characteristic polynomial,
for sol'ns to $L(y)=0$
for $L = y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1 y' + a_0$ a_j's <u>constant</u>

Most of our material is on Wednesday's notes.

An interesting tie-in between Euler's formula and addition angle formulas from trigonometry is the identity

$$e^{i(\alpha+\beta)} = e^{i\alpha}e^{i\beta}$$

<u>Exercise</u>: Show this identity is true, i.e. use Euler's formula to express both sides in terms of cosines & sines, and show this is a correct equality.
In practice, since you expect (hope) that the rules of exponents should apply for complex exponentials, this means that if you can remember Euler, you can recover the trig addition angle formulas.

Using the ideas we've been discussing, you get the following algorithm to find all sol'ns to $L(y)=0$ for the const. coeff. linear DE of order n:

<u>Theorem</u>: Let $L(y) = y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1 y' + a_0 y$ a_j const.

$p(r) = r^n + a_{n-1}r^{n-1} + \dots + a_1 r + a_0$

if r_j is a real root of p, $p(r_j)=0 \Rightarrow y = e^{r_j x}$ solves $L(y)=0$

if $(r-r_j)^{k_j}$ is a factor of $p \rightarrow$ $y_1 = e^{r_j x}$, $y_2 = x e^{r_j x}$, $\vdots$, $y_{k_j} = x^{k_j-1} e^{r_j x}$ are k_j lin ind sol'ns to $L(y)=0$

if $r_j = a+bi$ is a complex root, $p(a+bi)=0 \Rightarrow y_1 = e^{ax}\cos bx$, $y_2 = e^{ax}\sin bx$ are l.i. sol'ns to $L(y)=0$

if $(r-r_j)^{k_j}$ is a factor of p

also get $xe^{ax}\cos bx$, $xe^{ax}\sin bx$

$\vdots$

$x^{k_j-1}e^{ax}\cos bx$, $x^{k_j-1}e^{ax}\sin bx$

In this way we can always find a basis for $\ker(L)$, (assuming we can figure out how to factor p!)

<u>Exercise</u> Find the general solution to

$$y''' - 5y'' + 24y' - 20y = 0 \,!$$