Math 2280-1
Tuesday Sept. 23

$L(y) := y^{(n)} + a_{n-1}(x)y^{(n-1)} + \dots + a_1(x)y' + a_0(x)y$, each $a_j(x) \in C(I)$.

$L : C^n(I) \to C(I)$

summary so far!

Theorem 1 L is linear

Theorem 2 Let $f \in C(I)$, $x_0 \in I$, $\vec{b} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix} \in \mathbb{R}^n$. Then $\exists!$ sol'n to IVP

$$\text{IVP} \begin{cases} L(y) = f & \text{on } I \\ y(x_0) = b_0 \\ y'(x_0) = b_1 \\ \vdots \\ y^{(n-1)}(x_0) = b_{n-1} \end{cases}$$

(proof postponed, but we gave plausibility arguments).

Theorem 3 The general sol'n to $L(y) = f$ is $y = y_P + y_H$, where y_P is a particular sol'n and y_H is the general sol'n to the homogeneous DE $L(y) = 0$.

(general linear transformation fact)

Theorem 4 $\dim(\ker L) = n$

pf: We used $\exists!$ theorem (2) to exhibit a basis for $\ker L$ with n "vectors."
They were the functions $y_1, y_2, \dots y_n$ where y_j is the sol'n to

$$L(y_j) = 0$$
$$IV(y_j) = e_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow \text{entry } j \qquad IV(z(x)) := \begin{bmatrix} z(x_0) \\ z'(x_0) \\ \vdots \\ z^{(n-1)}(x_0) \end{bmatrix}.$$

so $IV(c_1y_1 + c_2y_2 + \dots + c_ny_n)$
$= IV(c_1y_1) + IV(c_2y_2) + \dots + IV(c_ny_n)$
$= c_1\vec{e}_1 + c_2\vec{e}_2 + \dots + c_n\vec{e}_n = \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

thus we can solve any IVP at x_0 with linear combos of $y_1 \dots y_n$.
So every sol'n to $L(y) = 0$ is a linear combo of $y_1 \dots y_n$!!

- so $\{y_1, \dots y_n\}$ span $\ker L$.
- if $c_1y_1 + \dots + c_ny_n \equiv 0$, then $IV(c_1y_1 + \dots + c_ny_n) = \vec{0}$
  so $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \vec{0}$, so $\{y_1, \dots y_n\}$ are ind

span & ind $\Rightarrow$ basis
$\Rightarrow \dim(\ker L) = n$ ∎

Please look this summary sheet over before class, and if Theorem 4 confuses you please try to formulate a question about specifically which claim is confusing. This is an amazing theorem actually, so it's O.K. to feel confused (at first.).
We'll work examples & finish Monday's notes after discussing your question