Math 2280-1
Tuesday Sept 2

On Friday we discussed the existence-uniqueness theorem for solutions to the IVP

$$\text{IVP.}\ \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

We used separable DE's to illustrate this theorem.

The text has some interesting examples too, in § 1.4

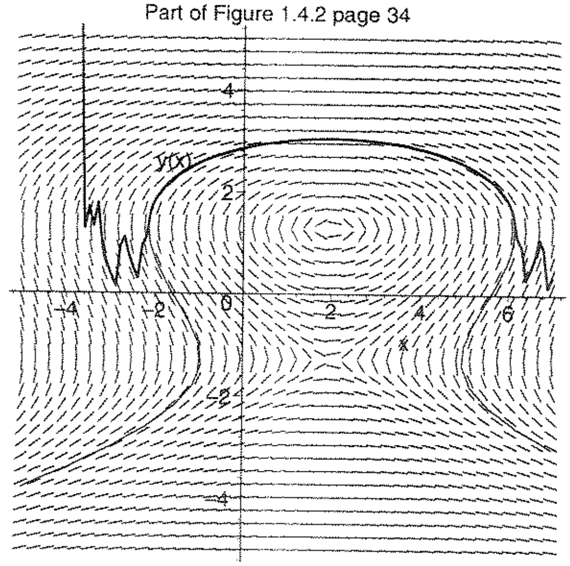
Exercise 1a): Solve the differential equation $\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$. For what IVP's does this DE have no $\exists$ sol'ns?

1b) Discuss ways in which the "implicit" solutions could be made explicit. Do all solutions show some symmetry?

1c) Discuss Maple output.

```
> with(DEtools):
> deqtn:=diff(y(x),x)=(4-2*x)/(3*y(x)^2-5):
  part1:=DEplot(deqtn,y(x),x=-5..7,y=-5..5,{[y(1)=3]},arrows=line,
     color=black,linecolor=black,dirgrid=[40,40],stepsize=.1,
     title='Part of Figure 1.4.2 page 34 '):
  with(plots):
  part2:=implicitplot(y^3-5*y=4*x-x^2+9,x=-5..7,y=-5..5,color=black)
  :
  display({part1,part2});
Warning, the name changecoords has been redefined
```

Part of Figure 1.4.2 page 34

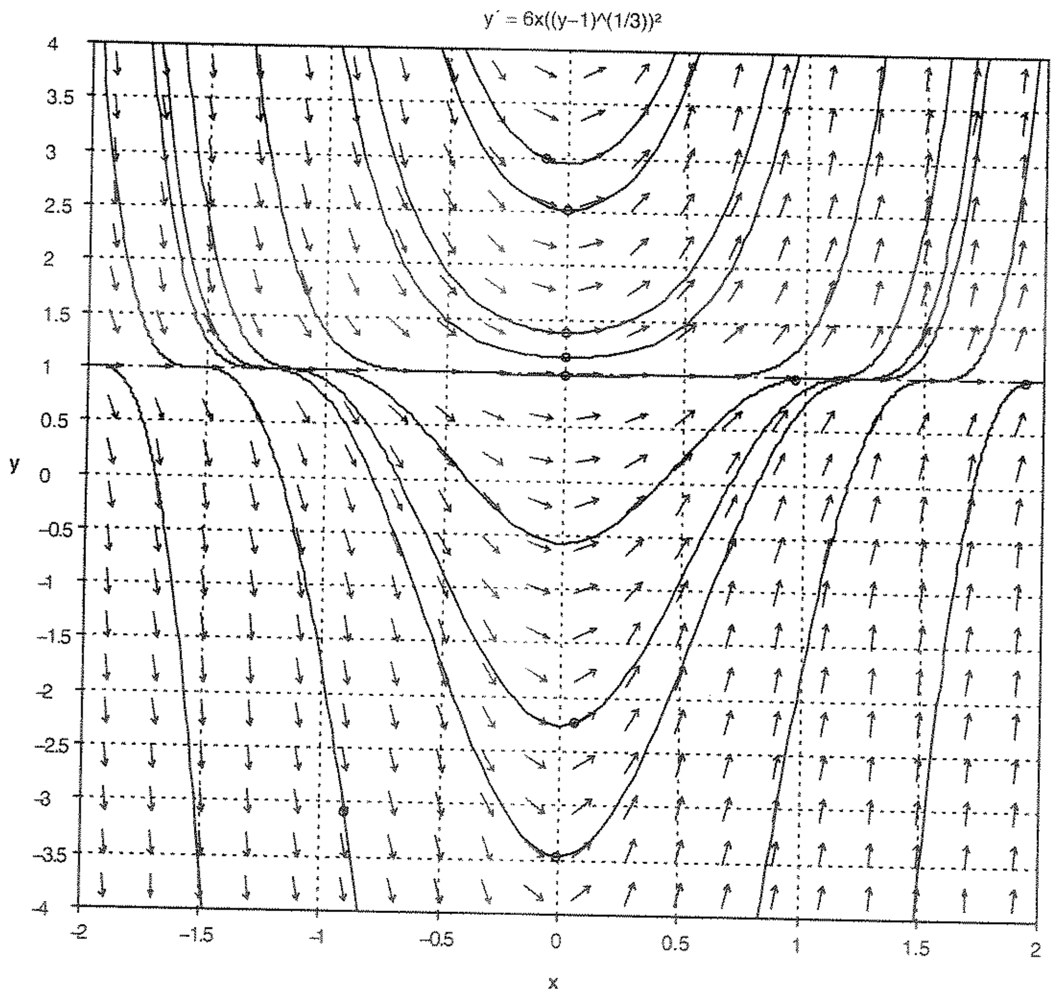


Exercise 2 (Example 4 p.36)

Find all solutions to the DE

$$\frac{dy}{dx} = 6x(y-1)^{2/3}$$

Notice the singular solutions!!

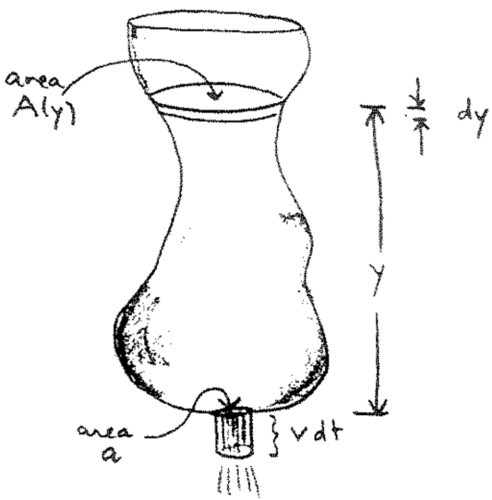


In text & hw are exponential growth/decay
+ Newton's law of cooling problems.

We did a problem like this last week ("dead body")
and the exponential growth/decay should be reminiscent of Calculus.

Let's do a separable DE application/experiment that might be new to you!

<u>Torricelli's Law</u> for draining tanks:



the speed v with which water leaves hole is

$$\boxed{v = \sqrt{2gy}}$$

reason: $KE + PE = \text{const}$

in a small time interval dt
a mass of water

$$dM = \rho\, dV = \rho A(y)\, dy$$

is lost from the top; replaced with equal mass

$$dM = \rho\, dV = \rho a v\, dt$$

shooting from bottom.

Since

loss in PE = gain in KE

$$\cancel{(dM)}\, g y = \tfrac{1}{2}\cancel{(dM)}\, v^2$$

$$v = \sqrt{2gy} \quad \blacksquare$$

We can express Torricelli as a separable DE by equating to two expressions for dM (or dV) on the right:

$$A(y)\, dy = a v\, dt$$

$$A(y)\, dy = a\sqrt{2gy}\, dt$$

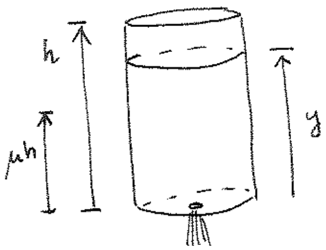
$$\boxed{A(y)\frac{dy}{dt} = -k\sqrt{y}}$$

separable DE!

Experiment!!

Cylindrical cistern: $A(y) = A$ const.

$$\frac{dy}{dt} = -k y^{1/2} \qquad \text{(different } k\text{)}.$$



Let T_μ be the amount of time it takes the cistern to empty from full height h, to height μh $(0 \le \mu \le 1)$

Show the time it takes to empty the tank is given by

$$\boxed{T_0 = \frac{T_\mu}{1-\mu^{1/2}}}$$

Nalgene bottle experiment:

I marked off the bottle so that we can use $\mu = .5$

So if we time how long it takes to empty half the height, and call it $T_{1/2}$, then the total time estimate will be

$$T_{0\,\cancel{tot}} = \frac{1}{1-\sqrt{.5}}\, T_{.5} \approx (3.41)\, T_{.5}$$

Experiment

$T_{1/2} =$

$(3.41)\, T_{1/2} =$

$T_0 =$