Math 2280-1
Monday Sept 15

<u>HW</u>: Maple project 1 (§1.5) is now due this Friday Sept 19.

<u>HW</u> for Monday Sept 22:

2.3 2 3, 9, 11, 13 14, 15 16 17 18

See Tuesday notes:

2.4 29 29a) by hand, 29bc probably with Maple 5

2.5 5, 25 In 25 also compare numerical answers to exact answers obtained by solving this IVP by hand

2.6 5 (the #5's in 2.4, 2.5, 2.6 are the same IVP via Euler, improved Euler and Runge Kutta

3.1 1, 2, 4, 5, 11, 13, 17, 27, 29, 30, 31, 33, 34, 35

3.2 2 5, 9, 11, 13, 21, 22 25 26

In calc: (and §1.2)

$\uparrow y$ $\quad m\frac{dv}{dt} = F_G = -mg$

$$\frac{dv}{dt} = -g$$

$$v = -gt + v_0$$

$$y = -\frac{1}{2}gt^2 + v_0 t + y_0$$

add resistance?

$$m\frac{dv}{dt} = F_G + F_f$$

$$|F_f| \cong k|v|^p \qquad 1 \le p \le 2 \quad \text{empiricle}$$

$$\boxed{\begin{array}{l}\underline{p=1} \quad (\text{"linear" model}) \\ m\frac{dv}{dt} = -mg - kv\end{array}}$$

force will cause an acceleration <u>opposite</u> to direction of motion

... this is low speed "linearization"

$$\text{if } F_f = F(v) = \underbrace{F(0)}_{0} + F'(0)v + \frac{1}{2!}F''(0)v^2 + \ldots$$

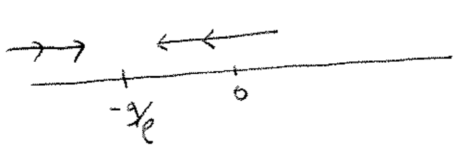
for $|v|$ small, "negligable"

$$\frac{dv}{dt} = -g - \rho v \qquad \rho := \frac{k}{m}$$

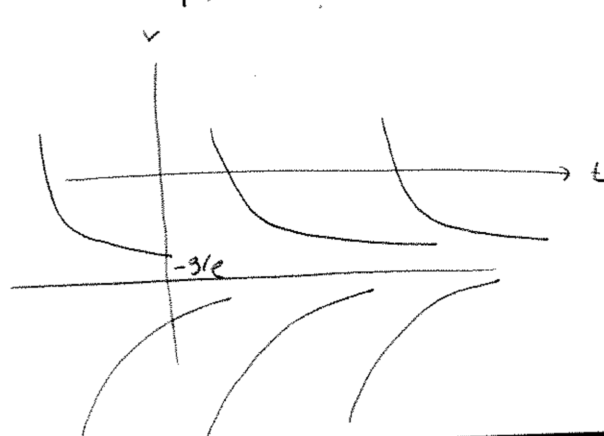
$$= -\rho\left(v + \frac{g}{\rho}\right)$$

equil soltn: $v = -g/\rho$

phase portrait:



slope field:



analytic sol'n

$$\frac{dv}{dt} = -\rho\left(v + \frac{g}{\rho}\right)$$

$$\frac{dv}{v + g/\rho} = -\rho\, dt$$

$$\ln|v + g/\rho| = -\rho t + C \qquad t = 0 \Rightarrow C = \ln|v_0 + g/\rho|$$

$$= -\rho t + \ln|v_0 + g/\rho|$$

$$|v + g/\rho| = |v_0 + g/\rho| e^{-\rho t}$$

$$v + g/\rho = (v_0 + g/\rho) e^{-\rho t} \qquad \text{(why?)}$$

$$\boxed{v = \underbrace{-g/\rho}_{v_T \text{ "terminal velocity}} + (v_0 + g/\rho) e^{-\rho t}}$$

$$y = \int v(t)\,dt = t v_T + \left(\frac{v_0 - v_T}{-\rho}\right) e^{-\rho t} + C \qquad y_0 = \frac{v_T - v_0}{\rho} + C$$

$$\text{so } C = y_0 + \frac{v_0 - v_T}{\rho}$$

$$\boxed{y = t v_T + \left(\frac{v_0 - v_T}{\rho}\right)\left(1 - e^{-\rho t}\right) + y_0}$$

Examples 1 & 2 p. 98-100

crossbow bolt

$v_0 = 49$ m/s

$g = 9.8$ m/s^2

$y_0 = 0$

no friction

$y = -4.9t^2 + 49t$

$v = -9.8t + 49$

max ht at $t = 5$ sec

$y_{max} = y(5) = 49(2.5)$

$= 122.5$ m

time aloft $= 10$ sec.

linear drag

$\rho = .04$ (drag coeff; empirical)

corresponds to

$|v_T| = \frac{g}{\rho} = \frac{9.8}{.04} = 245$ m/sec

(you could measure this to deduce ρ)

$v_0 - v_t = 49 + 245 = 294$

so

$v = -245 + 294 e^{-.04t}$

$v = 0$ at $\frac{245}{294} = e^{-.04t}$

$t_{max} \approx 4.56$ sec

$y = -245t + (294)(25)(1 - e^{-.04t})$

$y(t_{max}) = ?$

When does bolt hit ground?

Computation sheet for Example 2, section 2.3

Time of maximum height:

```
> 25*ln(294.0/245);
  solve(-245+294*exp(-.04*t)=0,t); #should be same
```

$$4.558038920$$

$$4.558038920$$

Formulas for height and velocity functions

```
> v:= t -> 294*exp(-.04*t) - 245;
```

$$v := t \to 294\,\mathrm{e}^{(-0.04\,t)} - 245$$

```
> y:= t-> -245*t + 294*25*(1 - exp(-.04*t));
```

$$y := t \to -245\,t + 7350 - 7350\,\mathrm{e}^{(-0.04\,t)}$$

Maximum height, return time tor ground time spent falling, landing speed:

```
> y(4.558038920); #max height
```

$$108.280465$$

```
> solve(y(t)=0,t);  #find when returns to ground
```

$$9.410949931,\ 0.$$

```
> 9.410949931 - 4.558038920; #time descending
```

$$4.852911011$$

```
> v(9.410949931); #speed when it lands
```

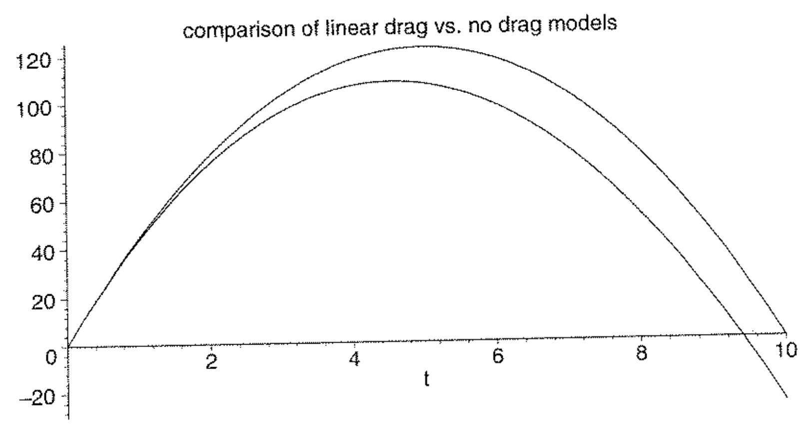
$$-43.2273093$$

Conclusions: bolt rises for 4.56 seconds, to a height of 108.3 meters. Then it spends 4.85 seconds descending, landing with a velocity of -43.3 meters per second.

```
> with(plots):
Warning, the name changecoords has been redefined
> z:= t->-4.9*t^2 + 49*t;  #the no resistance model
```

$$z := t \to -4.9\,t^2 + 49\,t$$

```
> plot({z(t),y(t)}, t = 0..10, color=black,
  title='comparison of linear drag vs. no drag models');
```



quadratic drag is also interesting...

<u>going up</u>:

$$m\frac{dv}{dt} = -mg - kv^2$$

$$\frac{dv}{dt} = -g\left(1 + \frac{f}{g}v^2\right)$$

$$\frac{dv}{1 + \frac{f}{g}v^2} = -g\,dt\ ..$$

arctan!

<u>going down</u>:

$$m\frac{dv}{dt} = -mg + kv^2$$

$$\frac{dv}{dt} = -g\left(1 - \frac{f}{g}v^2\right)$$

$$\frac{dv}{1 - \frac{f}{g}v^2} = \cancel{a} - g\,dt$$

parfrac! (or $\tanh^{-1}$)