Math 2280-1
Wed Sept 10

* Postpone §2.3 HW one week.
* Problem session tomorrow (LCB 225, 9:40-10:30)
* On Friday we shall meet in LCB 115, the computer classroom, so that you can work on your projects. I'll also explain how to convert $A\cos\omega t + B\sin\omega t$ into $C\cos(\omega t - \phi)$

At the end of yesterday we were considering

<u>autonomous</u> 1st order DE's: $\frac{dx}{dt} = f(x)$

<u>equilibrium sol'ns</u> of DE's:

<u>stability</u> of equilibria: $x = c$ stable:

$x = c$ unstable:

$x = c$ asymptotically stable (stronger than stable):
$\exists\, \delta > 0$ s.t. $|x(0) - c| < \delta \Rightarrow \lim_{t\to\infty} x(t) = c$
fix this in yesterday's notes

<u>Theorem</u> (More complete version of theorem on page 3 Tuesday notes)
Consider the aut. 1st order DE

$$\frac{dx}{dt} = f(x)$$

with $f(x)$ continuously differentiable in x.
Let $x = c$ be an equilibrium sol'n; $f(c) = 0$.
Then the following <u>local</u> phase portraits imply stability/instability, as indicated:
$c \in I$, some open interval

picture at right implied by $f(c) = 0$ and $f'(c) > 0$: $f<0$, $f>0$: $\leftarrow \leftarrow \underset{c}{\bullet} \rightarrow \rightarrow$ Unstable

$f'(c) < 0$: $f>0$, $f<0$: $\rightarrow \underset{c}{\bullet} \leftarrow$ Stable (asymptotically stable!)

$f>0$, $f>0$: $\rightarrow \underset{c}{\bullet} \rightarrow$ unstable (asymptotically stable from the left, unstable to the right)

$f<0$, $f<0$: $\leftarrow \underset{c}{\bullet} \leftarrow$ unstable (" " right, " left)

We might try proving the second of these (the stable case), on the back of this page! (Its proof is typical.)

Then,

- Discuss explosion/extinction model page 4 Tuesday notes & do example there.

Now, let's harvest a logistic population! (e.g. fisheries).

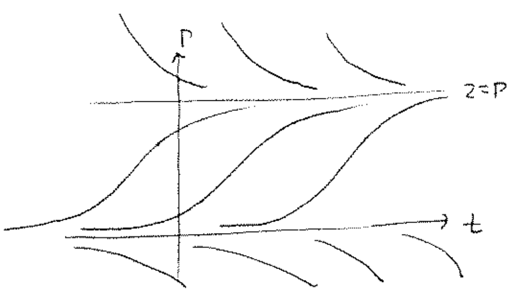
$$\frac{dP}{dt} = \underbrace{aP - bP^2}_{\text{logistic}} \; \underbrace{-\,h}_{\text{constant rate harvesting.}}$$

constant rate harvesting. (see §2.2 #24)

one could also consider a term $-hP$, which would maybe correspond to constant effort harvesting (see §2.2 #23)

e.g. $\frac{dP}{dt} = 2P - P^2$
$= P(2-P)$

vs $\frac{dP}{dt} = 2P - P^2 - h$



consider what happens for different h values

roots of RHS: $P^2 - 2P + h = 0$ has roots

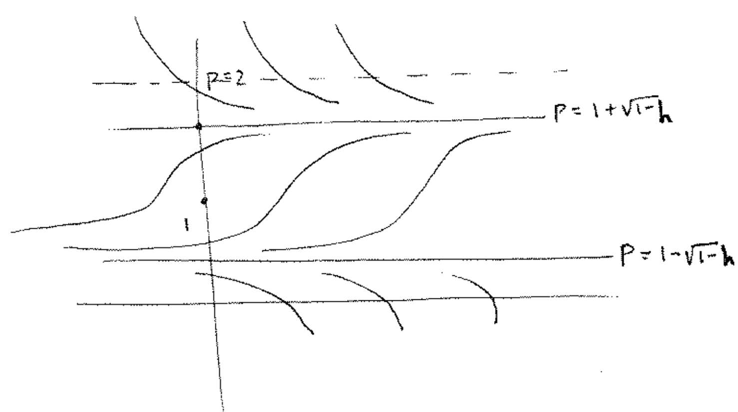
$$P = \frac{2 \pm \sqrt{4-4h}}{2} = 1 \pm \sqrt{1-h}$$

so, for $0 \le h \le 1$,

$$\frac{dP}{dt} = -\left(P - (1+\sqrt{1-h})\right)\left(P - (1-\sqrt{1-h})\right)$$

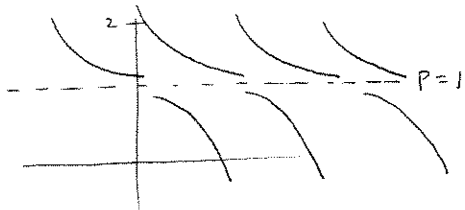
still looks logistic (& fishery won't collapse) if $h \ll 1$

extinction zone

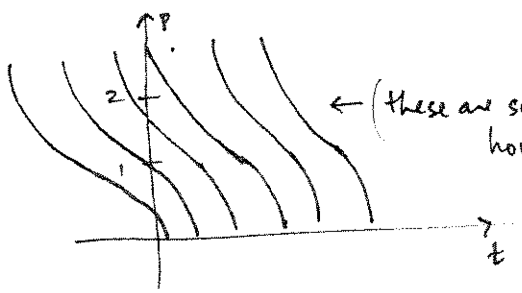


as $h \nearrow 1^-$ the middle zone (where populations increase to the modified carrying capacity) shrinks in width, until at $h=1$ the two roots coalesce;

$$\frac{dP}{dt} = 2P - P^2 - 1 = -(P^2 - 2P + 1) = -(P-1)^2$$



and for $h > 1$, $2P - P^2 - h = -(P^2 - 2P + h) = -((P-1)^2 + h-1) < 0 \quad \forall P$ (but least negative) @ $P=1$

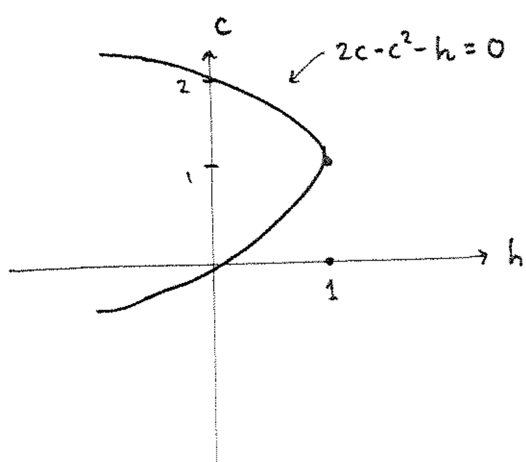


← (these are supposedly horizontal translations of each other)

Extinction $\forall P_0$!!

this model gives a plausible explanation for why fishery after fishery has collapsed around the world – if $h<1$ but near 1, and if something perturbs the system a little bit (e.g. increased fishing pressure, a big storm, etc.), you could be confronted with "sudden", unexpected fishery collapse.

"bifurcation diagram" of equilibrium sol'ns, in the h-c plane:



$h<0$
stocking!

$h>0$
harvesting