Math 2280-1
Monday Oct 27
§5.2 - 5.3

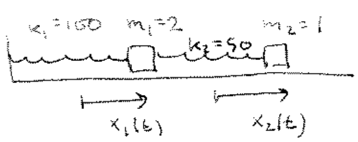
HW for Monday Nov. 3
5.2 2, 3, 9, 15, 27, 35 (you may use Maple for eigenvectors)
5.3 3, 7, 9, 14, 16, 17, 21 ← interesting!
Maple exploration §5.3 ... see paragraph at bottom of p. 331, "For you"
5.4 1, 7, 11, 29

discuss Theorem 1 p.2 Friday
(if $[A]_{n\times n}$ has n distinct evals λ_i,
with $A\vec{v}_i = \lambda_i$, then
$\{e^{\lambda_1 t}\vec{v}_1, e^{\lambda_2 t}\vec{v}_2, \ldots e^{\lambda_n t}\vec{v}_n\}$
is a basis for sol'n space to
$\vec{x}'(t) - A\vec{x} = \vec{0}$)

do example 3, p 4-5 Friday (Glucose-insulin)
with complex evals & evects, and
guess how Theorem 1 generalizes in case
of n distinct evals, but some of them complex.

Then begin §5.3 : Undamped spring systems

Example 1 p. 321



$$2x_1'' = -100x_1 + 50(x_2 - x_1)$$

$$1\,x_2'' = -50(x_2 - x_1)$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -150 & 50 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$M\vec{x}'' = K\vec{x}$$

in general, with n masses and up to $n+1$ springs, M & K are $n\times n$ matrices

$$\Rightarrow \quad \vec{x}'' = A\vec{x} \qquad \text{where } A = M^{-1}K.$$

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$ in this example.

What is the dimension of the solution space? (Careful!)

Look for solutions

$\vec{x}(t) = \cos\omega t\ \vec{v}$ & $\sin\omega t\ \vec{v}$, based on previous experience and conservation of energy:

if $e^{\lambda t}\vec{v}$ is a solution
Reλ must $=0$ or else
solution will either decay
or grow exponentially, violating
constant total energy!

$x' = -\omega\sin\omega t\ \vec{v}$

$x'' = -\omega^2\cos\omega t\ \vec{v}$

$A\vec{x} = \cos\omega t\ A\vec{v}$

must have

$A\vec{v} = -\omega^2\vec{v}$

$\vec{v}$ an eigenvector of A
with eval $\lambda = -\omega^2$.

- hope A has n lin ind evects.

  Each evect gives 2 l.i. solns $\cos\omega t\ \vec{v}$, $\sin\omega t\ \vec{v}$

  $\Rightarrow$ $2n$ lin ind. solns

  $\Rightarrow$ basis.

So, find general soln to

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Do you get

$$\vec{x}(t) = \underbrace{\left(c_1\cos 5t + c_2\sin 5t\right)}_{C_1\cos(5t-\alpha_1)}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \underbrace{\left(c_3\cos 10t + c_4\sin 10t\right)}_{C_2\cos(10t-\alpha_2)}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Explain what this solution means physically.
If you force this spring system, what angular frequencies for the forcing cosine fcn would induce resonance?