Math 2280-1
November 10, 2008
nonhomogeneous linear systems of differential equations -
variation of parameters

Work for example 4 page 367

```
[ > with(linalg):
[ > A:=matrix(2,2,[4,2,3,-1]);
    eigenvectors(A);
    exponential(A,s);
```

$$A := \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$[5, 1, \{[2, 1]\}], [-2, 1, \{[1, -3]\}]$$

$$\begin{bmatrix} \frac{1}{7}\mathbf{e}^{(-2s)} + \frac{6}{7}\mathbf{e}^{(5s)} & \frac{2}{7}\mathbf{e}^{(5s)} - \frac{2}{7}\mathbf{e}^{(-2s)} \\ \frac{3}{7}\mathbf{e}^{(5s)} - \frac{3}{7}\mathbf{e}^{(-2s)} & \frac{6}{7}\mathbf{e}^{(-2s)} + \frac{1}{7}\mathbf{e}^{(5s)} \end{bmatrix}$$

```
[ > x0:=matrix(2,1,[7,3]);
```

$$x0 := \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

```
[ > f:=t->evalm(-t*exp(-2*t)*matrix(2,1,[15,4]));
    #the nonhomogeneous term.
```

$$f := t \rightarrow \text{evalm}(-t\,\mathbf{e}^{(-2t)}\,\text{matrix}(2, 1, [15, 4]))$$

```
[ > f(s);
```

$$\begin{bmatrix} -15\,s\,\mathbf{e}^{(-2s)} \\ -4\,s\,\mathbf{e}^{(-2s)} \end{bmatrix}$$

```
[ > evalm(exponential(-A,s)&*f(s));
```

$$\begin{bmatrix} -15\left(\frac{6}{7}\mathbf{e}^{(-5s)} + \frac{1}{7}\mathbf{e}^{(2s)}\right)s\,\mathbf{e}^{(-2s)} - 4\left(-\frac{2}{7}\mathbf{e}^{(2s)} + \frac{2}{7}\mathbf{e}^{(-5s)}\right)s\,\mathbf{e}^{(-2s)} \\ -15\left(-\frac{3}{7}\mathbf{e}^{(2s)} + \frac{3}{7}\mathbf{e}^{(-5s)}\right)s\,\mathbf{e}^{(-2s)} - 4\left(\frac{1}{7}\mathbf{e}^{(-5s)} + \frac{6}{7}\mathbf{e}^{(2s)}\right)s\,\mathbf{e}^{(-2s)} \end{bmatrix}$$

```
[ > simplify(%);
```

$$\begin{bmatrix} -s\,(14\,\mathbf{e}^{(-7s)} + 1) \\ -s\,(-3 + 7\,\mathbf{e}^{(-7s)}) \end{bmatrix}$$

Maple (or I) currently has trouble directly evaluating the solution formula: I can't seem to make Maple integrate matrix valued functions without writing an explicit procedure (this did not used to be the case in older versions): The procedure below takes a matrix valued expression in s and returns a matrix

expression in t, for which each entry is the integral of the input from s=0 to s=t:

```
> Integratematrix:=proc(mat,m,n,s)
  local H, #matrix function to be returned
        i,  #row index
        j;  #column index

  H:=matrix(m,n):
   for i from 1 to m do
     for j from 1 to n do
     H[i,j]:=int(mat[i,j],s=0..t):
     od:
   od:
  return(evalm(H));
  end:
> integrand:=simplify(evalm(exponential(-A,s)&*f(s)));
     #the integrand in the solution formula
```

$$integrand := \begin{bmatrix} -s\,(14\,e^{(-7\,s)}+1) \\ -s\,(-3+7\,e^{(-7\,s)}) \end{bmatrix}$$

```
> Integratematrix(integrand,2,1,s);
```

$$\begin{bmatrix} 2\,t\,e^{(-7\,t)}+\dfrac{2}{7}\,e^{(-7\,t)}-\dfrac{t^2}{2}-\dfrac{2}{7} \\[2mm] \dfrac{3\,t^2}{2}+t\,e^{(-7\,t)}+\dfrac{1}{7}\,e^{(-7\,t)}-\dfrac{1}{7} \end{bmatrix}$$

```
> sol:=t->evalm(exponential(A,t)&*(x0+Integratematrix(integrand,2,1,
  s)));
     #solution formula
```

$$sol := t \rightarrow evalm(\text{`\&*`}(exponential(A,\,t),\,x0+Integratematrix(integrand,\,2,\,1,\,s)))$$

```
> simplify(sol(t));  #see page 368!!!
```

$$\begin{bmatrix} \dfrac{3}{7}\,e^{(-2\,t)}-\dfrac{1}{2}\,e^{(-2\,t)}\,t^2+\dfrac{46}{7}\,e^{(5\,t)}+2\,e^{(-2\,t)}\,t \\[3mm] \dfrac{23}{7}\,e^{(5\,t)}+e^{(-2\,t)}\,t-\dfrac{2}{7}\,e^{(-2\,t)}+\dfrac{3}{2}\,e^{(-2\,t)}\,t^2 \end{bmatrix}$$

```
>
```