Name: Solutions

I.D. number

Math 2280-1
FINAL EXAM
Dec 17, 2008

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. Laplace Transform Tables are included with this exam. **In order to receive full or partial credit on any problem, you must show all of your work and justify your conclusions.** This exam counts for 30% of your course grade. It has been written so that there are 200 points possible, however, and the point values for each problem are indicated in the right-hand margin. **Good Luck!**

problem	score	possible
1	______	35
2	______	40
3	______	20
4	______	20
5	______	25
6	______	20
7	______	40

1a) ~~A~~ motorboat containing the pilot has total mass of 360 kilograms, and its motor is able to ~~provide~~ 120 Newtons ~~of thrust. However~~, when the boat is in motion, drag from the water produces a force of 6 newtons for each meter/sec of boat velocity. Use Newton's law to explain why (while the motor is on) the boat velocity satisfies the differential equation

$$\frac{dv}{dt} = \frac{1}{3} - \frac{v}{60}$$

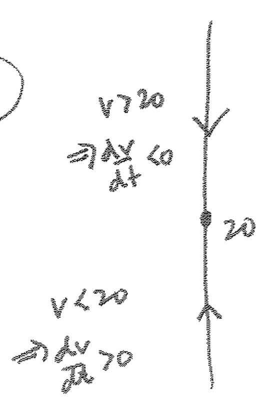
(5 points)

$$m\frac{dv}{dt} = \text{net forces}$$
$$= 120 - 6v$$
$$360\frac{dv}{dt} = 120 - 6v$$
$$\frac{dv}{dt} = \frac{1}{3} - \frac{v}{60}$$

1b) What is the equilibrium solution for the velocity v(t)? Is this solution stable or unstable? Explain with a phase diagram.

(5 points)

$\frac{dv}{dt} = 0$ when $\frac{1}{3} = \frac{v}{60} \Rightarrow v = 20$



so $v = 20$ is asymptotically stable

1c) Solve the initial value problem for the boat's velocity, assuming the boat starts at rest. Use the integrating-factor method we learned in Chapter 1,for first order linear differential equations.

(10 points)

$$\frac{dv}{dt} + \frac{1}{60} v = \frac{1}{3}$$

$$\left(e^{\frac{1}{60}t} v\right)' = \frac{1}{3} e^{\frac{1}{60}t}$$

$$e^{\frac{1}{60}t} v = \int \frac{1}{3} e^{\frac{1}{60}t} dt = 20 e^{\frac{1}{60}t} + C$$

$$v = 20 + Ce^{-\frac{1}{60}t}$$

$$v(0) = 0 = 20 + C \Longrightarrow C = -20$$

$$\boxed{v = 20 - 20e^{-\frac{1}{60}t}}$$

1d) Resolve the same IVP, this time using the algorithm for separable differential equations.

(10 points)

$$\frac{dv}{dt} = \frac{1}{3} - \frac{v}{60}$$

$$= -\frac{1}{60}(v-20)$$

$$\frac{dv}{v-20} = -\frac{1}{60} dt$$

int: $\ln|v-20| = -\frac{1}{60}t + C_1$

exp: $|v-20| = e^{C_1} e^{-\frac{1}{60}t}$

unabs $v - 20 = Ce^{-\frac{1}{60}t}$

$$v(0) = 0 \Rightarrow -20 = C.$$

$$\boxed{v = 20 - 20e^{-\frac{1}{60}t}}$$

1e) If the boat starts at rest, how long does it take to reach 75% of its terminal velocity?

(5 points)

$v_T = 20$ m/sec.

$.75 v_T = 15$

$$15 = 20 - 20e^{-\frac{1}{60}t}$$

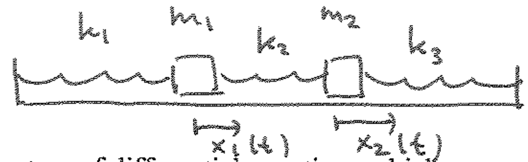
$$5 = 20e^{-\frac{1}{60}t}$$

$$4 = e^{\frac{1}{60}t}$$

$$\ln 4 = \frac{t}{60}$$

$$\boxed{t = (\ln 4)60 \text{ sec.}}$$

2) Consider a two mass, 3 spring system consisting of two masses with mass m_1 and m_2, and springs with Hooke's constants k_1, k_2, k_3, as sketched below. You can think of the masses as sliding along a frictionless table, or as hanging vertically. Let $x_1(t)$ and $x_2(t)$ measure displacement from equilbrium, as indicated. Assume there are no external forces.



2a) Derive the system of differential equations which governs the motion of the masses. (5 points).

$$m_1 x_1'' = k_2(x_2 - x_1) - k_1 x_1 \qquad = -(k_1+k_2)x_1 + k_2 x_2$$

$$m_2 x_2'' = -k_3 x_2 - k_2(x_2 - x_1) \qquad = k_2 x_1 + (-k_2 - x_3) x_2$$

2b) What is the dimension of the solution space to this system of DEs? Explain briefly. (5 points)

dim = 4.

sys is equiv. to a homog sys of 4 1^{st} order DE's.

2c) Consider the case in which the left mass is 2 kg and the right mass is 1 kg, and the left and right springs don't exist, and the middle spring has Hooke's constant 6 N/m. Show that the system in (2a) reduces to

$$\begin{bmatrix} \frac{d^2}{dt^2} x_1(t) \\ \frac{d^2}{dt^2} x_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(5 points)

from above: $k_1 = k_3 = 0$, $k_2 = 6$, $m_1 = 2$, $m_2 = 1$

matrix form

$$2x_1'' = -6x_1 + 6x_2 \qquad \div 2: \quad x_1'' = -3x_1 + 3x_2$$

$$1 x_2'' = 6x_1 - 6x_2 \qquad\qquad x_2'' = 6x_1 - 6x_2$$

2d) Find the general solution to the mass-spring system in (2c). For your convenience, the system is rewritten below.

$$\begin{bmatrix} \frac{d^2}{dt^2} x_1(t) \\ \frac{d^2}{dt^2} x_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(10 points)

$$\begin{vmatrix} -3-\lambda & 3 \\ 6 & -6-\lambda \end{vmatrix} = \lambda^2 + 9\lambda + 18 - 18 = \lambda(\lambda+9)$$

$\omega = \sqrt{-\lambda}$, except when $\lambda = 0$ get $(c_1 + c_2 t)\,\vec{v}$:

$\lambda = 0$

$$\left[\begin{array}{cc|c} -3 & 3 & 0 \\ 6 & -6 & 0 \end{array}\right] \qquad \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = -9$, $\omega = 3$

$$\left[\begin{array}{cc|c} 6 & 3 & 0 \\ 6 & 3 & 0 \end{array}\right] \qquad \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \vec{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

$$\boxed{(c_1 + c_2 t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (c_3 \cos 3t + c_4 \sin 3t)\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \vec{x}_H(t)}$$

2e) Describe the two fundmental motions (modes) for the system above, which superpose to give the general solution.

(5 points)

$(c_1 + c_2 t)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$: train moving at constant speed, with spring at equilibrium.

$(c_3 \cos 3t + c_4 \sin 3t)\begin{bmatrix} 1 \\ -2 \end{bmatrix}$: masses oscillating with $\omega = 3$ rad/sec, out of phase, with 2nd mass having twice the amplitude as the first.

$(c_3 \cos 3t + c_4 \sin 3t) = C\cos(3t - \alpha_1)$

2f) Now suppose there is an external sinusoidal force, so that the second order system becomes inhomogeneous,

$$\frac{d^2}{dt^2}x(t) = A\,x + \cos(\omega\,t)\,b\,.$$

Using matrix algebra (like we did in class, and you did in your Maple project), derive a formula for a particular solution

$$x_p(t) = \cos(\omega\,t)\,c$$

to this system. What happens to this solution if ω is close to one of the natural frequencies?

(10 points)

try $x_p = \cos\omega t\,\vec{c}$

$$\left.\begin{array}{l} x_p'' = -\omega^2\cos\omega t\,\vec{c} \\ A x_p + \cos\omega t\,\vec{b} = \cos\omega t\,A\vec{c} + \cos\omega t\,\vec{b} \end{array}\right\} \text{ equate. } \div \cos\omega t$$

$$-\omega^2\vec{c} = A\vec{c} + \vec{b}$$

$$-\vec{b} = (A+\omega^2 I)\,\vec{c}$$

$$\vec{c} = -(A+\omega^2 I)^{-1}\vec{b}$$

$(\omega \neq 0, 3)$.

(If $\omega = 0,3$ this form of $\vec{x}_p$ would not work, because system would resonate)

If ω is close to 0 or 3 it is likely that at least one entry in $\vec{c}$ is large, since you are close to resonance.

In fact, $$(A+\omega^2 I)^{-1} = \begin{bmatrix} -3+\omega^2 & 3 \\ 6 & -6+\omega^2 \end{bmatrix}^{-1} = \frac{1}{\omega^2(\omega^2-9)}\begin{bmatrix} -6+\omega^2 & -3 \\ -6 & -3+\omega^2 \end{bmatrix}$$

3a) Find e^{At} for the matrix from problem 2,

$$A = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix}.$$

(10 points)

$\lambda = 0 \qquad \lambda = -9$

$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

FMS for $\vec{x}' = A\vec{x}$:

$$\begin{bmatrix} 1 & e^{-9t} \\ 1 & -2e^{-9t} \end{bmatrix} \qquad FMS(0)^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}^{-1} = -\frac{1}{3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 1 & e^{-9t} \\ 1 & -2e^{-9t} \end{bmatrix} \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2+e^{-9t} & 1-e^{-9t} \\ 2-2e^{-9t} & 1+2e^{-9t} \end{bmatrix}$$

3b) Find e^{Bt} for the matrix

$$B = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

(10 points)

$$|B-\lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{vmatrix} = (\lambda-3)^2+16 = (\lambda-3+4i)(\lambda-3-4i)$$

$$\lambda = 3 \pm 4i$$

$\lambda = 3+4i$

$$\left[\begin{array}{cc|c} -4i & -4 & 0 \\ 4 & -4i & 0 \end{array}\right] \qquad \left[\begin{array}{cc|c} i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\vec{v} = \begin{bmatrix} +1 \\ -i \end{bmatrix}$$

$$e^{(3+4i)t}\begin{bmatrix} +1 \\ -i \end{bmatrix} = e^{3t}(\cos 4t + i\sin 4t)\begin{bmatrix} +1 \\ -i \end{bmatrix} = \begin{bmatrix} +e^{3t}\cos 4t + ie^{3t}\sin 4t \\ e^{3t}\sin 4t - ie^{3t}\cos 4t \end{bmatrix}$$

$$= \underbrace{e^{3t}\begin{bmatrix} \cos 4t \\ +\sin 4t \end{bmatrix}}_{\vec{x}_1(t)} + i\,\underbrace{e^{3t}\begin{bmatrix} +\sin 4t \\ -\cos 4t \end{bmatrix}}_{\vec{x}_2(t)}$$

$$@\ t=0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$FMS = e^{3t}\begin{bmatrix} \cos 4t & \sin 4t \\ +\sin 4t & -\cos 4t \end{bmatrix}$$

$$e^{At} = e^{3t}\begin{bmatrix} \cos 4t & \sin 4t \\ \sin 4t & -\cos 4t \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \boxed{e^{3t}\begin{bmatrix} \cos 4t & -\sin 4t \\ \sin 4t & \cos 4t \end{bmatrix}}$$

4) Use Laplace transform to solve the initial value problem for the undamped oscillator, where the system begins at rest. There are two cases,

4a) Driving angular frequency ω is not the natural frequency ω_0:

$$\frac{d^2}{dt^2}x(t) + \omega_0^2 x(t) = F_0 \cos(\omega t)$$
$$x(0) = 0$$
$$D(x)(0) = 0$$

(10 points)

$$s^2 X(s) + \omega_0^2 X(s) = F_0 \frac{s}{s^2+\omega^2}$$

$$\| $$

$$X(s)(s^2+\omega_0^2)$$

$$X(s) = F_0 s \left(\frac{1}{(s^2+\omega^2)(s^2+\omega_0^2)} \right) = F_0 s \left[\frac{1}{s^2+\omega^2} - \frac{1}{s^2+\omega_0^2} \right] \frac{1}{\omega_0^2-\omega^2}$$

$$= \frac{F_0}{\omega_0^2-\omega^2} \left[\frac{s}{s^2+\omega^2} - \frac{s}{s^2+\omega_0^2} \right]$$

So $\boxed{x(t) = \frac{F_0}{\omega_0^2-\omega^2}\left[\cos\omega t - \cos\omega_0 t \right]}$

4b) $\omega = \omega_0$:

$$\frac{d^2}{dt^2}x(t) + \omega_0^2 x(t) = F_0 \cos(\omega_0 t)$$
$$x(0) = 0$$
$$D(x)(0) = 0$$

(10 points)

$$s^2 X(s) + \omega_0^2 X(s) = F_0 \frac{s}{s^2+\omega_0^2}$$

$$X(s) = F_0 \frac{s}{(s^2+\omega_0^2)^2}$$

$$\boxed{x(t) = F_0 \frac{t}{2\omega_0} \sin\omega_0 t}$$

5a) Use your work from problem (4) to solve the initial value problem

$$\frac{d^2}{dt^2}x(t) + x(t) = 4\cos(3t) + 7\cos(t)$$

$$x(0) = 5$$

$$D(x)(0) = 2$$

notice that initial condition map is linear too, so

(10 points)

$$\boxed{x(t) = \underbrace{5\cos t + 2\sin t}_{\substack{x_H(t) \\ x_H(0)=5 \\ x_H'(0)=2}} + \underbrace{\frac{4}{1-9}(\cos 3t - \cos t) + \frac{7t}{2}\sin 2t}_{\substack{x(0)=0 \\ x'(0)=0}}}$$

$$\left(= \frac{19}{2}\cos t + 2\sin t - \frac{1}{2}\cos 3t + \frac{7}{2}t\sin 2t \right)$$

5b) In problem 6 you will verify that the 2π-periodic "tent" function has Fourier series

$$tent(t) = \frac{1}{2}\pi - \frac{4\left(\sum_{n=odd}\frac{\cos(nt)}{n^2}\right)}{\pi}.$$

Use this fact to find a particular solution to

$$\frac{d^2}{dt^2}x(t) + x(t) = 2\,\underbrace{tent(t)}.$$

$$\pi - \frac{8}{\pi}\sum_{n\,odd}\frac{\cos nt}{n^2}$$

(10 points)

by superposition

$n=1$: resonates

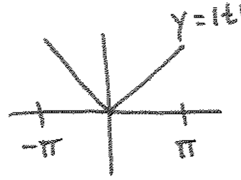
$$x_p(t) = \underbrace{\pi}_{\substack{x_p \text{ for} \\ RHS=\pi}} - \frac{8}{\pi}\sum_{\substack{n\ odd \\ n\geq 3}}\underbrace{\frac{1}{n^2}\frac{1}{(1-n^2)}\cos nt}_{4a} - \underbrace{\frac{8}{\pi}\frac{t}{2}\sin t}_{n=1 \text{ term}}$$

5c) Does the solution to forced oscillation problem in (5b) exhibit resonance? Explain.

(5 points)

Yes

6a) Let $tent(t)$ be the 2π periodic extension of absolute value function from the interval $[-\pi, \pi]$ to the entire real line. Prove that $tent(t)$ has Fourier Series given by



$$tent(t) = \frac{1}{2}\pi - \frac{4\left(\sum_{n=odd} \frac{\cos(n\,t)}{n^2}\right)}{\pi}.$$

(12 points)

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt = \frac{2}{\pi}\int_0^{\pi} t\cos nt\,dt \quad \text{because } |t| \text{ is even}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{2}{\pi}\int_0^{\pi} t\,dt = \frac{2}{\pi}\frac{\pi^2}{2} = \pi$$

$b_n = 0$ since $|t|$ is even.

$$\frac{2}{\pi}\int_0^{\pi} \underset{\substack{\uparrow\\ u\\ du=dt}}{t}\, \underbrace{\cos nt\,dt}_{\substack{dv\\ v=\frac{\sin nt}{n}}} = \frac{2}{\pi}\left[\overbrace{\left. t\,\cancel{\frac{\sin nt}{n}}\right]_0^{\pi}}^{0} - \int_0^{\pi}\frac{\sin nt}{n}\,dt\right] = -\frac{2}{\pi}\left[-\frac{\cos nt}{n^2}\right]_0^{\pi}$$

$$= \left.\frac{2}{\pi}\frac{\cos nt}{n^2}\right]_0^{\pi} = \begin{cases} \frac{2}{\pi n^2}(-1-1) & n \text{ odd} \\ \frac{2}{\pi n^2}(1-1) & n \text{ even}\end{cases} = \begin{cases} -\frac{4}{\pi}\frac{1}{n^2} & n \text{ odd} \\ 0 & n \text{ even}\end{cases}$$

So

$$tent(t) \sim \frac{a_0}{2} + \sum a_n\cos nt = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n\,odd}\frac{1}{n^2}\cos nt$$

6b) We were allowed to use the = sign for the Fourier series in (6a) because the tent function is continuous and piecewise differentible, so the Fourier series converges to the function at each point. Use this fact to prove the magic formulas (use the first to prove the second):

$$\sum_{n=odd}\frac{1}{n^2} = \frac{1}{8}\pi^2$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{6}\pi^2$$

(8 points)

@ $t=0$ $\quad tent(0) = 0 = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n\,odd}\frac{1}{n^2}$

So $\quad \sum_{n\,odd}\frac{1}{n^2} = \frac{\pi}{2}\cdot\frac{\pi}{4} = \frac{\pi^2}{8}$ ✓

$$S = \sum_{n=1}^{\infty}\frac{1}{n^2} = \sum_{n\,odd} + \sum_{n\,even} = \frac{\pi^2}{8} + \underbrace{\sum_{k=1}^{\infty}\frac{1}{(2k)^2}}_{=\frac{1}{4}\sum\frac{1}{k^2}} = \frac{\pi^2}{8} + \frac{1}{4}S$$

thus $\frac{3}{4}S = \frac{\pi^2}{8}$

$S = \frac{\pi^2}{8}\cdot\frac{4}{3} = \frac{\pi^2}{6}$ ✓

7) Consider the system of differential equations below which models two populations x(t) and y(t):

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 9x - x^2 - 2xy \\ \underbrace{12y - y^2}_{\text{logistic}} - \underbrace{2xy}_{\text{competition}} \end{bmatrix}$$

7a) If this was a model of two interacting populations, what kind would it be? Explain.

(2 points)

competition: either species is logistic by itself. but the presence of either one hurts the other because the xy coeffs are both negative

7b) Find all four equilbrium solutions to this system of differential equations. (Hint: One of them is [5, 2].)

(8 points)

$x(9-x-2y)=0$

$y(12-y-2x)=0$

$x=0$: $y=0$ gives $\begin{bmatrix}0\\0\end{bmatrix}$; $12-y-2x=0$, $y=12$ gives $\begin{bmatrix}0\\12\end{bmatrix}$

$9-x-2y=0$: $y=0$, $x=9$ gives $\begin{bmatrix}9\\0\end{bmatrix}$; $12-y-2x=0$ gives $\begin{bmatrix}5\\2\end{bmatrix}$.

$x+2y=9$

$2x+y=12$

$$x = \frac{\begin{vmatrix} 9 & +2 \\ 12 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & +2 \\ 2 & 1 \end{vmatrix}} = \frac{-15}{-3} = 5$$

$$y = \frac{\begin{vmatrix} 1 & 9 \\ 2 & 12 \end{vmatrix}}{-3} = \frac{-6}{-3} = 2$$

7c) Find the linearization of the population model near the equilibrium solution [5,2]. Use eigenvalues for the linearization to classify the type of singularity in the nonlinear problem. For your convenience, the system is repeated below:

(10 points)

$$\begin{bmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 9x - x^2 - 2xy \\ 12y - y^2 - 2xy \end{bmatrix}$$

$$J(x,y) = \begin{bmatrix} 9-2x-2y & -2x \\ -2y & 12-2y-2x \end{bmatrix}$$

10

$$|J-\lambda I| = \begin{vmatrix} -5-\lambda & -10 \\ -4 & -2-\lambda \end{vmatrix}$$

$$= (\lambda+2)(\lambda+5) - 40$$

$$= \lambda^2 + 7\lambda - 30$$

$$= (\lambda+10)(\lambda-3)$$

$\lambda = -10, +3$

Saddle (unstable)

$$J(5,2) = \begin{bmatrix} 9-10-4 & -10 \\ -4 & 12-4-10 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -10 \\ -4 & -2 \end{bmatrix}.$$

So system is $\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} -5 & -10 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$

7d) Finding and using the eigenvectors from the linearization above, write the general solution $\begin{bmatrix} u(t) \\ v(t) \end{bmatrix}$ of the linearized problem at [5,2].

(10 points)

$\lambda = -10$

$$\left[\begin{array}{cc|c} 5 & -10 & 0 \\ -4 & 8 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\lambda = +3$

$$\left[\begin{array}{cc|c} -8 & -10 & 0 \\ -4 & -5 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

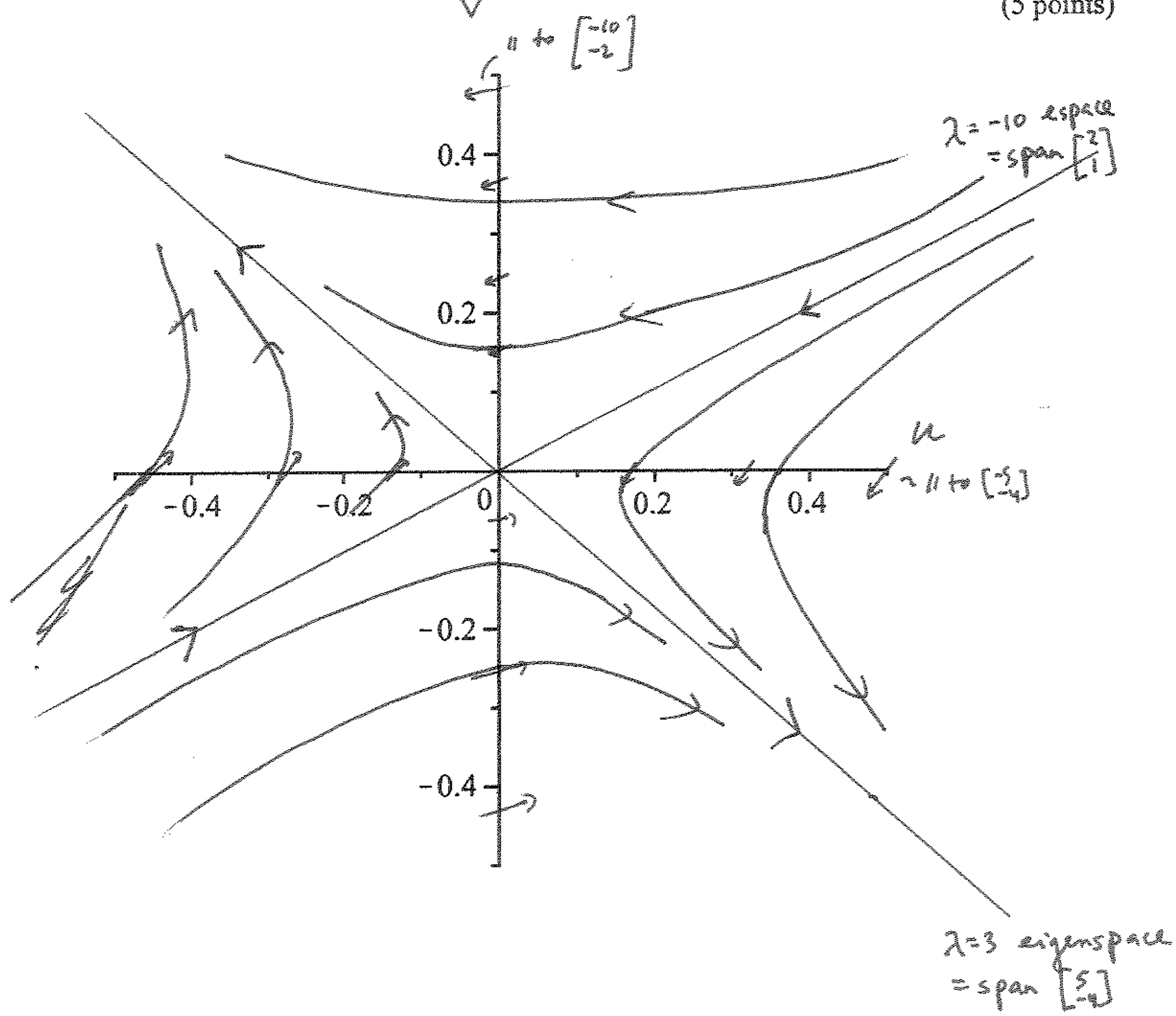
$$\vec{v} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = c_1 e^{-10t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

7e) Sketch the phase portrait for the linearized problem, using your work in (7d). Hint: the eigendirections should appear in your sketch.

(5 points)

✓



$$\begin{bmatrix}u'\\v'\end{bmatrix} = \begin{bmatrix}-5 & -10\\-4 & -2\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}$$

$$\begin{bmatrix}u\\v\end{bmatrix} = c_1 e^{-10t}\begin{bmatrix}2\\1\end{bmatrix} + c_2 e^{3t}\begin{bmatrix}5\\-4\end{bmatrix}.$$

7f) Use your work from (7e) to fill the portion of the phase portrait below which has been excised. Then describe what happens to solutions to the initial value problem for this system of DE's, depending on initial populations. You may wish to sketch representative solution curves, and indicate regions of the first quadrant in order to help you answer this part.

(5 points)

too much competition! $(b_1 b_2 < a_1 c_2)$

If int. pops point is below the dotted curve (separatrix), $y(t)$ dies out & $x(t) \to 9$ as $t \to \infty$.

If they're above it, then $x(t)$ dies out, and $y(t) \to 12$