Math 2280-1
MAPLE
1.3-1.4: separable DE's, slope fields, existence and uniqueness.
Friday August 29, 2008

Homework for Friday September 5:   (Hand in underlined problems.)

Maple Introduction for Math 2250, linked from our home page. The purpose of this assignment is to introduce you to Maple, its help features, some useful commands, and its ability to create documents like these class notes. If you have not used Maple before, or want a refresher, I highly recommend you complete this assignment (even though you're in Math 2280), which is to read and work through the notes for the Introduction to Maple sessions we are holding for all sections of Math 2250. (This work will not be graded.)

1.3: 3, **6**, 10, **11, 12, 13, 18, 21, 23, 29.** In problem 6, consider the actual problem to be **6a**. Then also do

  6b) verify $y(x) = x + C \, \mathbf{e}^{(-x)}$ solves the DE, and convince yourself the curves you sketch on the slope field are consistent with this formula.
  6c) Use dfield to draw the indicated solution graphs. (For 6a you may use dfield to draw the solution field and then add the solution curves by hand.)
  In problem 29, consider the book problem to be **29a**. Then also do
  29b) Get dfield to illustrate several of the non-unique solutions you verify exist in 29a.
Recall you can google the applet "dfield", but also that if you're using Math Department system the on-line applet won't print hard copies. We've made a copy of the applet you can call from a terminal window which will print hard copies. This copy is also called dfield.).
1.4: **9, 12, 19, 22,** 35, **41, 43**, **46**, **54, 66**.
1.5: 1, **7, 13, 34**, 36, **38, 41**.

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   I created today's notes using the software package Maple, to show you the kind of documents it is possible to create with Maple. Although this software has been produced by a private company since 1988, it originated with Canadian government support at the University of Waterloo; the originators used the leaf on the Canadian flag to motivate the name "Maple."

   You will be using Maple in Math 2280, in order to solve problems and do computer projects. You should think of this software as analgous to the service provided by Microsoft Word as a tool for writing papers - except Maple lets you create documents which combine text and mathematics. Alternately, you may use Maple to generate mathematical output, which you can then export to other documents. (That is closer to how engineers use Matlab, which is more efficient than Maple at doing extremely large computations, but lacks Maple's ability to create text documents and to easily do symbolic mathematical work.)

   I will use Maple 8 in Math 2280, because it has superior graphing capabilites over the more recent versions, which are Maple 11 and Maple 12. The main advantage of these latter versions is that they include more Microsoft Word type bells and whistles. Any Maple 8 document should open in Maple 11-12, but if you are using these latter Maple versions and you expect to reopen your document in Maple

8, you must save your file in the "classic" mode. Student computers around campus (Engineering, Marriott, Math Department, Heritage Commons) have Maple software installed, and in most places you can find both the earlier and latter versions. The University bookstore sells a student version of Maple 12 ($130), which you may wish to purchase at some point, although it is not at all necessary to do so for Math 2280.

When you open any of these Maple versions you can find a "new user's tour" in a "help" window, and I recommend you take this tour to get an overview of Maple's capabilities. From then on, you will learn to use Maple the same way you learned to use Microsoft Word, i.e. by using it, using the help features, and asking friends, lab assistants, TAs and teachers when you get stuck. We will also be running Maple introductions sessions specifically for Math 2250, but you could easily sneek into one of these. The currently scheduled sessions (we may add more) are:

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Saturday August 30: PCLab 1735, in Marriott MMC,   2-3 pm.
Tuesday September 2: LCB 115, 11:50-12:40 and 2:30-3:20
Wednesday September 3: LCB 115, 10:45-11:55.

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These sessions are first-come, first-serve, and about 30 people can take a session at a time.

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We will use today's notes to continue our discussion in Wednesday's notes of slope fields and of the existence-uniqueness theorem, section 1.3. Yesterday we reminded ourselves why the separation of variables "differential magic" gives correct (implicitly defined) solutions to separable first order DEs, section 1.4 I still had to go through that discussion with my 2250 class, and started writing these notes to do that. I decided to leave the discussion in our version - so you can see that Maple is able to display math equations, probably more nicely that MSWord, but not as nicely as "latex", which is the typsetting program mathematicians use to write their research papers.

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<u>Separable Differential Equations</u>

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A first order differential equation

$$\frac{dy}{dx} = h(x, y)$$

is called **separable** iff h(x,y) is a product of a function of x times a function of y,

$$\frac{dy}{dx} = g(x)\,\phi(y)\,.$$

This is equivalent to the DE

$$\frac{dy}{dx} = \frac{g(x)}{f(y)}\,,$$

where f and ϕ are reciprocal functions.

Solving separable DEs:

The algorithm is very simple, but magic: treat dy/dx as a quotient of differentials (?!), and multiply through to rewrite the DE as

$$f(y)\,dy = g(x)\,dx\,.$$

Then antidifferentiate the left side with respect to y and the right side with respect to x:

$$\int f(y)\,dy = \int g(x)\,dx\,.$$

If F(y) and G(x) are antiderivatives of f(y) and g(x), respectively, and if you collect the constants of integration on one side of the equation, then this yields an equation involving the solution function y, and the variable x:

$$F(y) = G(x) + C\,.$$

This equation defines y implicitly as a function of x. Sometimes you can use algebra to explicitly solve for y. The constant C can be adjusted to solve initial value problems.

Mathematical justification for the method of separating differentials:

The use of differentials is disguising an application of the chain rule. Here is the explanation for the magic method: The differential equation

$$\frac{dy}{dx} = \frac{g(x)}{f(y)}$$

can be rewritten without differential magic, as

$$f(y)\left[\frac{dy}{dx}\right] = g(x).$$

If y(x) is any solution to this rewritten equation, then the left side, namely

$$f(y(x))\left[\frac{dy}{dx}\right]$$

is the derivative with respect to x of

$$F(y(x))\,,$$

where F(y) is any antiderivative of f(y) (with respect to y). This is just the chain rule! Thus if G(x) is any antiderivative of g(x) (w.r.t.x), we can legally antidifferentiate the rewritten DE with respect to x (on both sides) to get

$$F(y(x)) = G(x) + C$$

which is what we got by differential magic before!

    Now dig out your Wednesday notes (or if you're online, open them in a separate tab)....we were about to begin

Exercise 3, page 3 Wednesday Aug 29 notes: Do parts 3a), 3b), 3c). I had intended to paste in a dfield picture of the slope field into Wednesday's notes, so that we could check our work.....but since I'm in Maple right now, I may as well check our work here!

Here's how you can use Maple to check your answer (once you find the right commands):

```
[ > with(DEtools):   #a package of differential equation commands
[ > dsolve([diff(y(x),x)=1+y(x)^2,y(0)=y[0]]);
```

$$y(x) = \tan(x + \arctan(y_0))$$

Maple can also draw slope fields and solution graphs, although the commands are more cumbersome than in the applet "dfield":

```
[ > deqtn:=diff(y(x),x)=1+y(x)^2:  #this is the DE
    DEplot(deqtn,y(x),-10..10,{[y(-8)=0],[y(-4)=0],
      [y(0)=0],[y(4)=0],[y(8)=0]},
      y=-10..10,arrows=line, color=black,linecolor=black,
      dirgrid=[30,30],stepsize=.1, title='don't go off on a
    tangent');
```


don't go off on a tangent
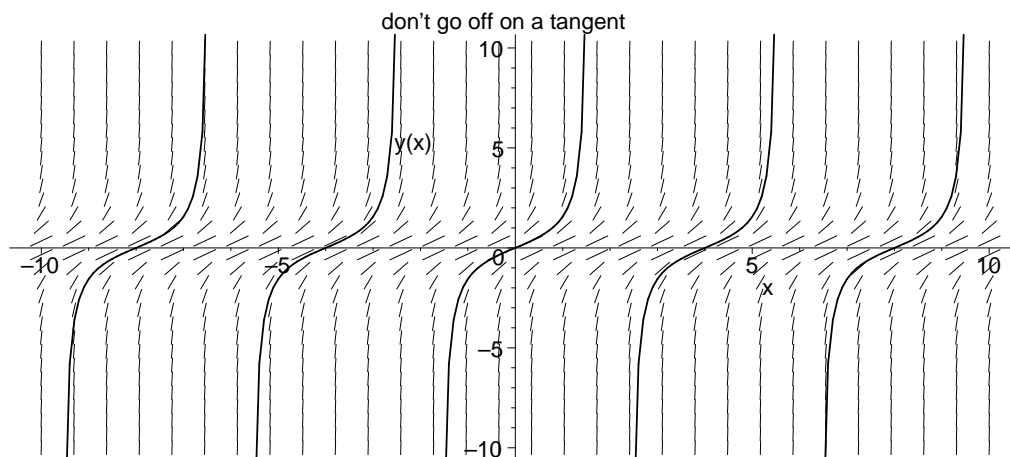
Exercise 4, page 4 Wednesday notes. We were to find solutions to
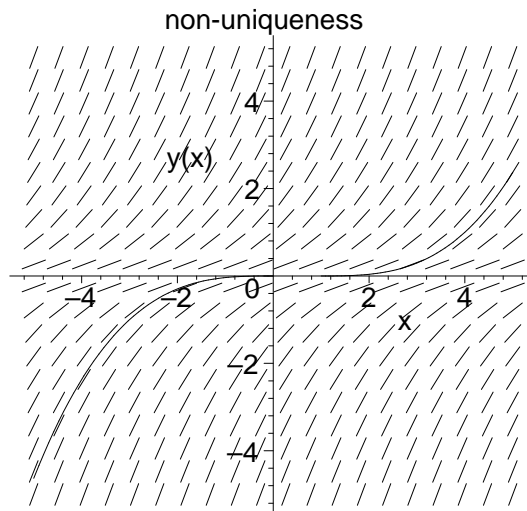
$$\frac{dy}{dx} = y^{(2/3)}$$

using separation of variables. What happens when Maple tries solving this DE symbolically? Does it yield all solutions?

```
> diffeqtn4:=diff(y(x),x)=y(x)^(2/3):
  dsolve(diffeqtn4,y(x));
```

$$y(x)^{(1/3)} - \frac{x}{3} - \_C1 = 0$$

Indeed, did separation of variables give us all the solutions?

For example, is does the following graph look like the graph of a solution, even though it's not one we found?

non-uniqueness

Finally, discuss the existence-uniqueness theorem on page 5 of Wednesday notes, and see how it is illustrated in the preceding exercises.