Math 2280-1
Wed 27 Aug. §1.2-1.3

optional problem session LCB 225 9:40-10:30
§1.1-1.2 tomorrow Thurs.

part of next weekhw is
§1.3 3, 6, 10, 11, 12, 13, 18, 21, 23, 29

also
6b) verify
$y(x) = x + Ce^{-x}$
solves the DE,
and convince yourself
the curves you sketch
on the slope field
are consistent with
this formula

6c) Use dfield
to draw the indicated
solution graphs
(for 6a) you can use
dfield to just draw
the slope field)

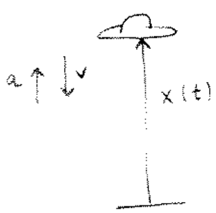
29b)
get dfield
to draw some
of these
solutions

- finish slope field construction, example 2 Tuesday

- hop back to §1.2 to do lunar lander problem p.13: (Exercise!)

lunar lander descending to moon at speed 450 m/s
when (additional) retro rockets are fired
they provide a deceleration of 2.5 m/s^2.

At what height should they be fired so that $v=0$ exactly when landing happens?



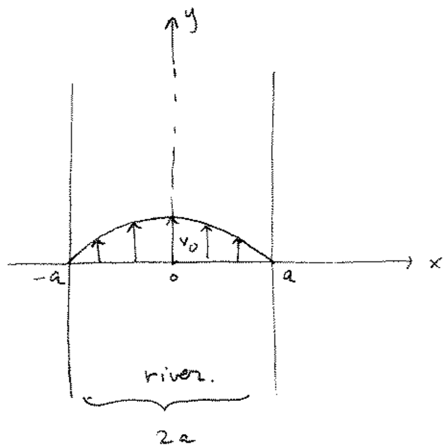
units

	English	metric
Force	lb	N
mass	slug	kg
distance	ft	m
time	s	s
g	32 ft/sec^2	9.8 m/s^2

g: accel of gravity, on earth

see conversion discussion p. 14.

swimmer problem (Examples 3-4 p.15-16). (Exercise 2)



$v_R(x) = v_0\left(1 - x^2/a^2\right)$ river velocity

swimmer starts at $(-a, 0)$
and swims due East with const veloc v_s
(aims)

where does swimmer land

$\frac{dx}{dt} = v_s$

$\frac{dy}{dt} = v_R$

2a)
Find the DE:

2b) If river is 1 mile wide and
$v_0 = 9$ mi/h
$v_s = 3$ mi/h

swimmer starts at $(-a, 0)$.
where does swimmer hit opposite shore?

Return to §1.3

well, actually, we'll also use this from §1.4:

"Recall" a separable DE can be written as

$$\frac{dy}{dx} = f(x)\,\phi(y) = \frac{f(x)}{g(y)}$$

math soltn

$$g(y)\frac{dy}{dx} = f(x)$$

$$\frac{d}{dx}\,G(y(x)) = \frac{d}{dx}\,F(x) \qquad \text{where } G(y) \text{ is antider of } g(y),\ F(x) \text{ is antideriv of } f(x)$$

$$\boxed{G(y) = F(x) + C}$$

expresses y <u>implicitly</u> as a fcn of x

sep of variables algorithm

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$\boxed{G(y) = F(x) + C}$$

You <u>may</u> be able to solve this equation <u>explicitly</u> for $y = y(x)$

<u>Exercise</u> 3a) Use separation of variables to solve $\frac{dy}{dx} = 1 + y^2$

3b) Draw some sol'n graphs onto the slope field

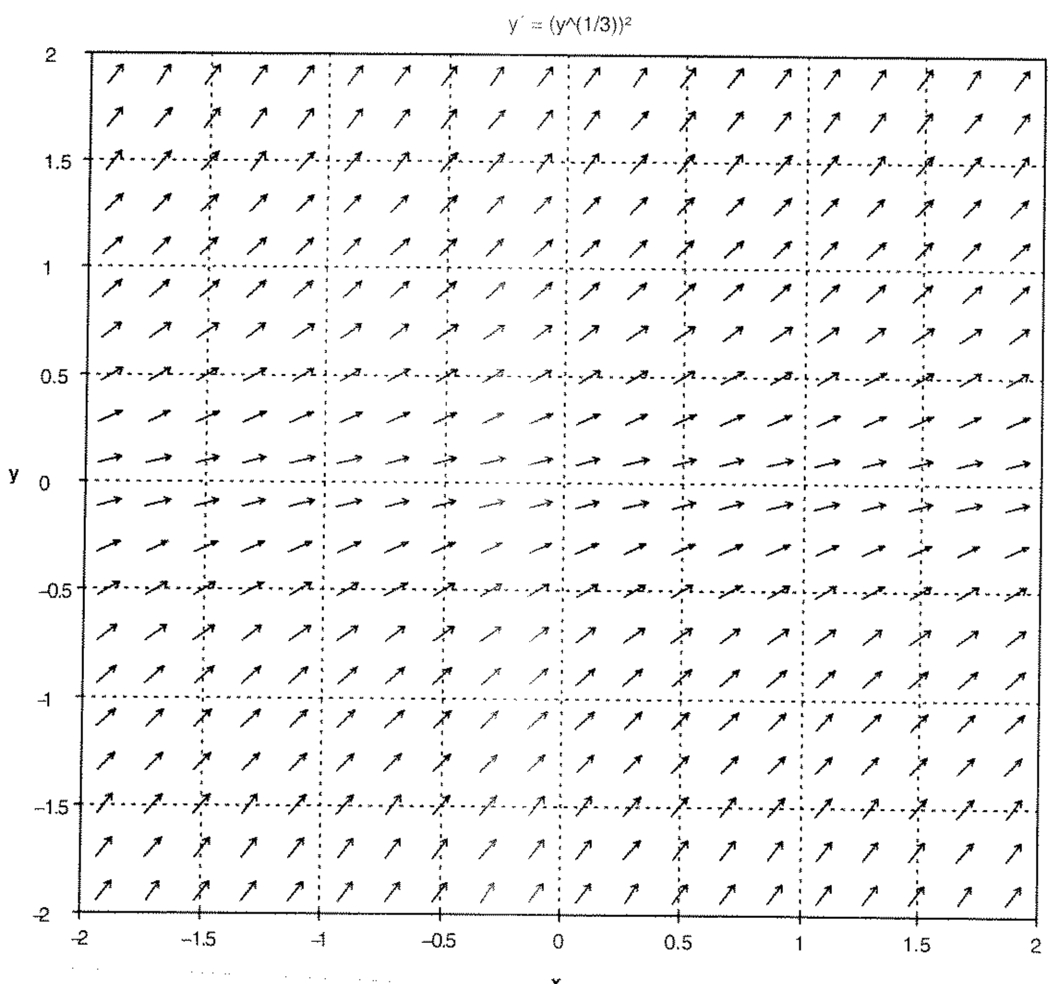
3c) Explain why each IVP has a sol'n, but this sol'n ~~need~~ does not exist for all x.

Exercise 4

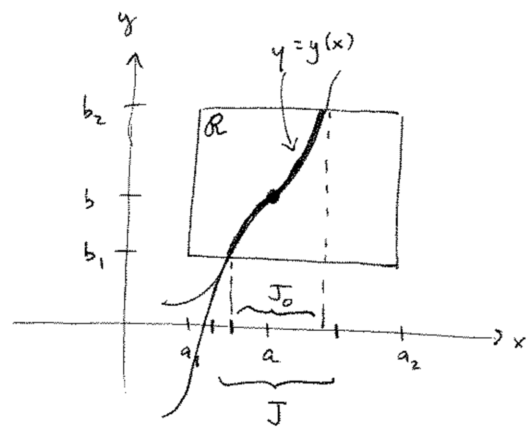
4a) Solve $\begin{cases} \dfrac{dy}{dx} = y^{2/3} \\ y(0) = 0 \end{cases}$

use separation of variables. Notice this IVP has lots of sol'ns.

4b Sketch some of these onto the slope field.



Usually, existence and uniqueness hold



Theorem 1 $\exists!$: (page 23)

Consider

$$\text{IVP} \begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

Let (a,b) be interior to a coord rectangle $\mathcal{R}$
$\left(\begin{matrix} a_1 \le x \le a_2 \\ b_1 \le y \le b_2 \end{matrix} \right)$

Let $f(x,y)$ be continuous in $\mathcal{R}$.

Then $\exists$ sol'n to IVP on some interval J containg a in its interior

If $\frac{\partial f}{\partial y}$ exists and is continuous in $\mathcal{R}$ then this solution is unique on any subinterval J_0 s.t. the solution graph lies inside $\mathcal{R}$.

for proof, see appendix

Exercise 5 Discuss how $\exists!$ theorem applies to Exercises 3, 4