2 linear equations in 2 unknowns:

$$a_{11}\,x + a_{12}\,y = b_1$$
$$a_{21}\,x + a_{22}\,y = b_2$$

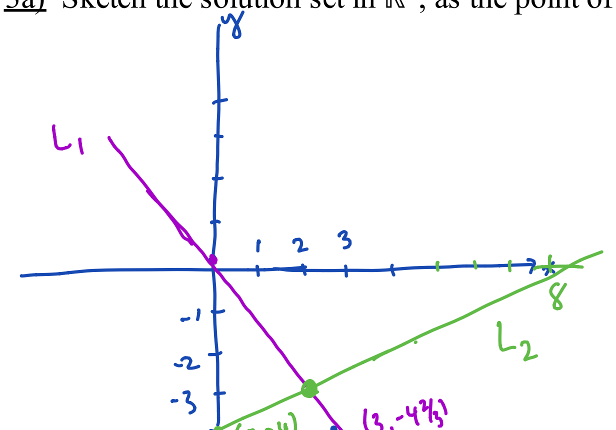
goal: find all $[x, y]$ making both of these equations true at once. Since the solution set to each single equation is a line one geometric interpretation is that you are looking for the intersection of two lines.

Exercise 3: Consider the system of two equations E_1, E_2:

$15 + 3y = 1$
$3y = -14$

$$E_1 \qquad 5\,x + 3\,y = 1$$ ← $(0, \tfrac{1}{3})$, $(3, -4\tfrac{2}{3})$
$$E_2 \qquad x - 2\,y = 8$$ $(8, 0)$

3a) Sketch the solution set in $\mathbb{R}^2$, as the point of intersection between two lines.



3b) Use the following three "elementary equation operations" to systematically reduce the system E_1, E_2 to an equivalent system (i.e. one that has the same solution set), but of the form

$$1\,x + 0\,y = c_1$$
$$0\,x + 1\,y = c_2$$

(so that the solution is $x = c_1, y = c_2$). Make sketches of the intersecting lines, at each stage.

The three types of elementary equation operation are below. Can you explain why the solution set to the modified system is the same as the solution set before you make the modification?

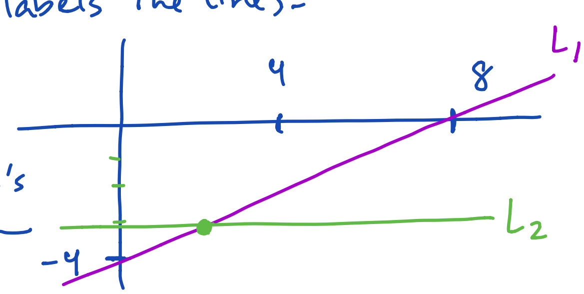
- interchange the order of the equations
- multiply one of the equations by a non-zero constant
- replace an equation with its sum with a multiple of a different equation.

$\boxed{\text{HW} \quad 1.1 \ \# \ 1, 3, 13, 17}$

$E_2 \to E_1$ $\qquad x - 2y = 8$ $\qquad$ to get $1 \cdot x$ in 1^{st} eqtn
$E_1 \to E_2$ $\qquad 5x + 3y = 1$ $\qquad$ just relabels the lines.

$x - 2y = 8$

$-5E_1 + E_2 \to E_2$ $\qquad 13y = -39$ $\qquad$ eliminate x's in 2nd eqtn



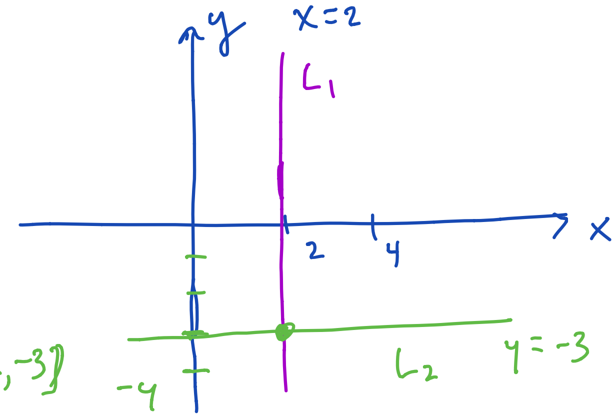
$$x - 2y = 8$$

$$\frac{E_2}{13} \to E_2 \qquad y = -3$$

lines are the same. We do this step to make it easy to eliminate the y's in eq'n 1

$$2E_2 + E_1 \to E_1 \qquad x = 2$$

$$y = -3$$



sol'n:

$[x, y] = [2, -3]$

Since elementary eq'n operations don't change the solution set to a system of linear eq'ns, $[x, y] = [2, -3]$ is the sol'n to original system:

$x - 2y = 8$: $2 + 6 = 8$ ✓

$5x + 3y = 1$: $10 - 9 = 1$ ✓

3c) Look at your work in 3b. Notice that you could have save a lot of writing by doing this computation "synthetically", i.e. by just keeping track of the coefficients and right-side values. Using R_1, R_2 as symbols for the rows, your work might look like the computation below. Notice that when you operate synthetically the "elementary equation operations" correspond to "elementary row operations":

- interchange two rows
- multiply a row by a non-zero number
- replace a row by its sum with a multiple of another row.

more complete notation:

$$\left[\begin{array}{cc|c} 5 & 3 & 1 \\ 1 & -2 & 8 \end{array}\right]$$

$R_2 \to R_1$
$R_1 \to R_2$

$$\begin{array}{c} R_2 \\ R_1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 5 & 3 & 1 \end{array}\right]$$

$-5R_1 + R_2 \to R_2$

$$\begin{array}{c} \\ -5R_1+R_2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 13 & -39 \end{array}\right]$$

$\frac{R_2}{13} \to R_2$

$$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -3 \end{array}\right]$$

$2R_2 + R_1 \to R_1$

$$\begin{array}{c} 2R_2+R_1 \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right] \longrightarrow \begin{array}{l} x = 2 \\ y = -3 \end{array} \ !$$

3d) What are the possible geometric solution sets to 1, 2, 3, 4 or any number of linear equations in two unknowns?

empty set (distinct parallel lines)

single point (all lines have a common point of intersection)

line (all lines are actually the same line)

all of $\mathbb{R}^2$ (all eqtns are $0x + 0y = 0$)

Math 2270-004
Tues Jan 9

- 1.1 Systems of linear equations
- 1.2 Row reduction and echelon form

Announcements:

- finish Monday's notes
- quiz tomorrow, §1.1, 1.2

Warm-up Exercise: was something like:

a) Do the lines

$E_1 \quad x - 3y = 6$

$E_2 \quad -2x + 6y = -4$

intersect?

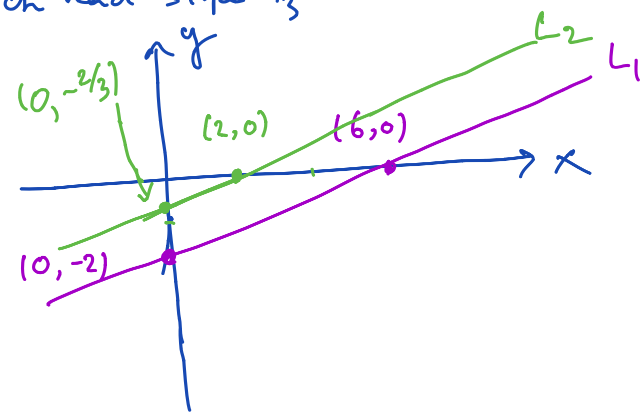
b) Explain.

these lines were parallel
(the each had slope $\frac{1}{3}$)

$$\left[\begin{array}{cc|c} 1 & -3 & 6 \\ -2 & 6 & -4 \end{array}\right]$$

$$\begin{array}{r} \\ 2R_1 + R_2 \rightarrow R_2 \end{array} \left[\begin{array}{cc|c} 1 & -3 & 6 \\ 0 & 0 & 8 \end{array}\right] \leftarrow \text{says } 0x + 0y = 8$$

no solutions!
so the original system has no solutions
"inconsistent"
lines don't intersect!



Continuing our discussion of solution sets to systems of equations from yesterday:

<u>Solutions to linear equations in 3 unknowns:</u>

What is the geometric question we're answering in these cases?

<u>Exercise 1)</u> Consider the system

$$\begin{aligned} x + 2\,y + z &= 4 \\ 3\,x + 8\,y + 7\,z &= 20 \\ 2\,x + 7\,y + 9\,z &= 23\,. \end{aligned}$$

Use elementary equation operations (or if you prefer, elementary row operations in the synthetic version) to find the solution set to this system. There's a systematic way to do this, which we'll talk about today. It's called <u>Gaussian elimination.</u> (Check Wikipedia about Gauss - he was amazing!)

<u>Hint:</u> The solution set is a single point, $[x, y, z] = [5, -2, 3]$.

$$\begin{array}{r|ccc|c} & 1 & 2 & 1 & 4 \\ & 3 & 8 & 7 & 20 \\ & 2 & 7 & 9 & 23 \end{array}$$

$$\begin{array}{r|ccc|c} & 1 & 2 & 1 & 4 \\ -3R_1+R_2\to R_2 & 0 & 2 & 4 & 8 \\ -2R_1+R_3\to R_3 & 0 & 3 & 7 & 15 \end{array}$$

$$\begin{array}{r|ccc|c} & 1 & 2 & 1 & 4 \\ R_2/2 \to R_2 & 0 & 1 & 2 & 4 \\ & 0 & 3 & 7 & 15 \end{array}$$

$$\begin{array}{r|ccc|c} & 1 & 2 & 1 & 4 \\ & 0 & 1 & 2 & 4 \\ -3R_2+R_3\to R_3 & 0 & 0 & 1 & 3 \end{array}$$

$$\begin{array}{r|ccc|c} -R_3+R_1\to R_1 & 1 & 2 & 0 & 1 \\ -2R_3+R_2\to R_2 & 0 & 1 & 0 & -2 \\ & 0 & 0 & 1 & 3 \end{array}$$

$$\begin{array}{r|ccc|c} -2R_2+R_1\to R_1 & 1 & 0 & 0 & 5 \\ & 0 & 1 & 0 & -2 \\ & 0 & 0 & 1 & 3 \end{array} \longrightarrow \begin{array}{l} x = 5 \\ y = -2 \\ z = 3 \end{array}$$