Syllabus for Math 2270-004 Differential Equations and Linear Algebra

Spring 2018

Instructor Professor Nick Korevaar
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office LCB 204, 801.581.7318
office hours M 2:00-3:00 p.m. LCB 204, T 4:30-6:00 location ~~TBA~~ JWB 240, and by appointment. Also available after class (briefly). (may change)
Lecture MTWF 12:55-1:45 p.m. MWF in WEB L110, T in WEB L102
don't forget

Course websites

Daily lecture notes and weekly homework assignments will be posted on our public home page.

http://www.math.utah.edu/~korevaar/2270spring18 up by tonight, hopefully

There are blank spaces in the notes where we will work out examples and fill in details together. Research has shown that class attendance with active participation - including individual and collaborative problem solving, and writing notes by hand - are effective ways to learn class material for almost everyone. Passively watching a lecture is not usually effective. Class notes will be posted at least several days before we use them, so that you have ample time to print them out. Printing for math classes is free in the Math Department Rushing Student Center, in the basement of LCB. Beyond what's outlined in the notes, there will often be additional class discussion related to homework and other problems.

Grades and exam material will be posted on our CANVAS course page; access via Campus Information Systems.

Textbook *Linear Algebra and its Applications*, 5th edition, by David D. Lay. ISBN: 032198238X

Final Exam logistics: Monday, April 30, 1:00-3:00 p.m., in our MWF classroom WEB L110. This is the University scheduled time and location.

Catalog description for Math 2270: Euclidean space, linear systems, Gaussian elimination, determinants, inverses, vector spaces, linear transformations, quadratic forms, least squares and linear programming, eigenvalues and eigenvectors, diagonalization. Includes theoretical and computer lab components.

Course Overview: This course is the first course in a year long sequence (2270-2280) devoted to linear mathematics. In this course, we study two objects: vectors and matrices. We start by thinking of vectors and matrices as arrays of numbers, then we progress to thinking of vectors as elements of a vector space and matrices as linear transformations. In our study of vectors and matrices, we learn to solve systems of linear equations, familiarize ourselves with matrix algebra, and explore the theory of vector spaces. Some key concepts we study are determinants, eigenvalues and eigenvectors, orthogonality, symmetric matrices, and quadratic forms. Along the way we encounter applications of the material in science and engineering. Students who continue into Math 2280 will see that linear algebra is one of the foundations, together with Calculus, upon which the study of differential equations is based.

Prerequisites: C or better in MATH 2210 or MATH 1260 or MATH 1321 or MATH 1320. Practically speaking, you are better prepared for this course if your grades in the prerequisite courses were above the "C" level.

Students with disabilities: The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, 581-5020. CDS will work with you and the instructor to make arrangements for accommodations. All information in this course can be made available in alternative format with prior notification to the Center for Disability Services.

Grading

Math 2270-004 is graded on a curve. By this I mean that the final grading scale may end up lower than the usual 90/80/70% cut-offs. **note:** In order to receive a grade of at least "C" in the course you must earn a grade of at least "C" on the final exam. Typical grade distributions in Math 2270 end up with grades divided roughly in thirds between A's, B's, and the remaining grades. Individual classes vary.

Details about the content of each assignment type, and how much they count towards your final grade are as follows:

- Homework (15%): There will be one homework assignment each week. Homework problems will be posted on our public page, and homework assignments will be due in class on Wednesdays. Homework assignments must be stapled. Unstapled assignments will not receive credit. I understand that sometimes homework cannot be completed on time due to circumstances beyond your control. To account for this, each student will be allowed to turn in **three** late homework assignments throughout the course of the semester. These assignments cannot be turned in more than one week late, and must be turned in on a Wednesday with the next homework assignment. You do not need to tell me the reason why your homework assignment is late.

- Quizzes (10%): At the end of most Wednesday classes, a short 1-2 problem quiz will be given, taking roughly 10 minutes to do. The quiz will cover relevant topics from the week's lectures, homework, and food for thought work. Two of a student's lowest quiz scores will be dropped. There are no makeup quizzes. You will be allowed and encouraged to work together on these quizzes.

- Midterm exams (30%): Two class-length midterm exams will be given, On Friday February ~~16~~ 9 and Friday March ~~30~~ 16. I will schedule a room for review on the Thursday before each midterm, at our regular class time of 12:55-1:45 p.m. No midterm scores are dropped.

- Final exam (30%): A two-hour comprehensive exam will be given at the end of the semester. As with the midterms, a practice final will be posted. Please check the final exam time, which is the official University scheduled time. It is your responsibility to make yourself available for that time, so make any arrangements (e.g., with your employer) as early as possible.

- Food for Thought questions (15%): On Fridays without exams, we will spend half of the class time working on a collection of thought provoking problems. These problems will lend themselves naturally to discussion, and students will work with their learning communities to discuss/debate/ponder these questions. Students will turn in their Friday Food for Thought responses the following Monday in class, and these will be graded for completion. Solutions to Friday Food for Thoughts will be posted on Canvas. Your lowest Friday Food for Thought score will be dropped. (In other words, you can miss one Friday Food for Thought without penalty.)

Strategies for success

- Attend class regularly, and participate actively.

- Read or skim the relevant text book sections and lecture note outlines *before* you attend class.

- Ask questions and become involved.

- Plan to do homework daily; try homework on the same day that the material is covered in lecture; do not wait until just before homework and lab reports are due to begin serious work.

- Form study groups with other students.

Learning Objectives for 2270

Computation vs. Theory: This course is a combination of computational mathematics and theoretical mathematics. By theoretical mathematics, I mean abstract definitions and theorems, instead of calculations. The computational aspects of the course may feel more familiar and easier to grasp, but I urge you to focus on the theoretical aspects of the subject. Linear algebra is a tool that is heavily used in mathematics, engineering, and science, so it will likely be relevant to you later in your career. When this time comes, you will find that the computations of linear algebra can easily be done by computing systems such as Matlab, Maple, Mathematica or Wolfram alpha. But to understand the significance of these computations, a person must understand the theory of linear algebra. Understanding abstract mathematics is something that comes with practice, and takes more time than repeating a calculation. When you encounter an abstract concept in lecture, I encourage you to pause and give yourself some time to think about it. Try to give examples of the concept, and think about what the concept is good for.

The essential topics

Be able to find the solution set to linear systems of equations systematically, using row reduction techniques and reduced row echelon form - by hand for smaller systems and using technology for larger ones. Be able to solve (linear combination) vector equations using the same methods, as both concepts are united by the common matrix equation $A\mathbf{x} = \mathbf{b}$.

Be able to use the correspondence between matrices and linear transformations - first for transformations between $\mathbb{R}^n$ and $\mathbb{R}^m$, and later for transformations between arbitrary vector spaces.

Become fluent in matrix algebra techniques built out of matrix addition and multiplication, in order to solve matrix equations.

Understand the algebra and geometry of determinants so that you can compute determinants, with applications to matrix inverses and to oriented volume expansion factors for linear transformations.

Become fluent in the language and concepts related to general vector spaces: linear independence, span, basis, dimension, and rank for linear transformations. Understand how change of basis in the domain and range effect the matrix of a linear transformation.

Be able to find eigenvalues and eigenvectors for square matrices. Apply these matrix algebra concepts and matrix diagonalization to understand the geometry of linear transformations and certain discrete dynamical systems.

Understand how orthogonality and angles in $\mathbf{R}^2, \mathbf{R}^3$ generalize via the dot product to $\mathbf{R}^n$, and via general inner products to other vector spaces. Be able to use orthogonal projections and the Gram-Schmidt process, with applications to least squares problems and to function vector spaces.

Know the spectral theorem for symmetric matrices and be able to find their diagonalizations. Relate this to quadratic forms, constrained optimization problems, and to the singular value decomposition for matrices. Learn some applications to image processing and/or statistics.

Week-by-Week Topics Plan

Topic schedule is subject to slight modifications as the course progresses, but exam dates are fixed.

Week 1: 1.1-1.3; systems of linear equations, row reduction and echelon forms, vector equations.

Week 2: 1.4-1.6; matrix equations, solution sets of linear systems, applications.

Week 3: 1.7-1.9; linear dependence and independence, linear transformations and matrices.

Week 4: 2.1-2.3; matrix algebra and inverses, invertible matrices.

Week 5: 2.4-2.5; partitioned matrices and matrix factorizations. **Midterm exam 1 on Friday February 9** covering material from weeks 1-5.

Week 6: 3.1-3.3; determinants, algebraic and geometric properties and interpretations.

Week 7: 4.1-4.2, 2.8; vector spaces and subspaces, nullspaces and column spaces, general linear transformations.

Week 8: 4.3-4.6; linearly independent sets, bases, coordinate systems, dimension and rank

Week 9: 4.7, 5.1-5.2; change of basis, eigenvectors and how to find them.

Week 10: 5.3-5.4; diagonalization, eigenvectors and linear transformations. **Midterm exam 2 on Friday March 16** covering material from weeks 6-10

Week 11: 5.5-5.6, 6.1-6.2; complex eigendata, discrete dynamical systems, introduction to inner products and orthogonality.

Week 12: 6.3-6.5; orthogonal projections, Gram-Schmidt process, least squares problems.

Week 13: 6.7-6.8, 7.1; general inner product spaces and applications, diagonalization of symmetric matrices

Week 14: 7.2-7.4; quadtratic forms, constrained optimization, singular value decomposition.

Week 15: 7.5; applications to image processing and statistics.

Week16: **Final exam Monday April 30, 1:00 - 3:00 p.m. in classroom WEB L110. This is the University scheduled time.**

· 20 minutes syllabus
· rest of class

Math 2270-004 Week 1 notes

We will not necessarily finish the material from a given day's notes on that day. Or on an amazing day we may get farther than I've predicted. We may also add or subtract some material as the week progresses, but these notes represent an outline of what we will cover. These week 1 notes are for sections 1.1-1.3.

Monday January 8:

- Course Introduction
- 1.1: Systems of linear equations

• Go over course information on syllabus and course homepage:

http://www.math.utah.edu/~korevaar/2270spring18

• Note that there is a quiz this Wednesday on the material we cover today and tomorrow. Your first homework assignment will be due next Wednesday, January 17.

Then, let's begin!

• What is a linear equation? What is a system of linear equations?

any equation in some number of variables $x_1, x_2, \ldots x_n$ *which can be written in the form*

$$a_1\, x_1 + a_2\, x_2 + \ldots + a_n\, x_n = b$$

where b and the coefficients $a_1, a_2, \ldots a_n$ *are real or complex numbers usually known in advance.*

Question: Why do you think we call an equation like that $\boxed{\text{"linear"?}}$

one reason: in 2 variables, the collection of all solutions is a line.
i.e. $ax + by = c$

another: variables appear in a "linear" way, i.e. as multiples of each variable, to the 1st power

Exercise 1) Which of these is a linear equation?

1a) For the variables x, y

$$3\,x + 4\,y = 6$$

yes

1b) For the variables s, t

$$2\,t = 5 - \sqrt{3}\,s$$

yes

1c) For the variables x, y

no

$$2\,x = 5 - 3\sqrt{y}$$

variables only appear with power 1

- Then the general linear system (LS) of m equations in the n variables $x_1, x_2, \ldots x_n$ can be written as

$$\text{LS}\left\{\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \quad\vdots \qquad\qquad\qquad\qquad\quad \vdots \quad\; \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{array}\right.$$

2 : 2nd eqn
n : multiplying n^{th} variables

In such a linear systems the coefficients a_{ij} and the right-side number b_j are usually known. The goal is to find values for the vector $\underline{\boldsymbol{x}} = [x_1, x_2, \ldots x_n]$ of variables so that all equations are true. (Thus this is often called finding "simultaneous" solutions to the linear system, because all equations will be true at once.)

Definition *The solution set of a system of linear equations is the collection of all solution vectors to that system.*

Notice that we use two subscripts for the coefficients a_{ij} and that the first one indicates which equation it appears in, and the second one indicates which variable its multiplying; in the corresponding *coefficient matrix A*, this numbering corresponds to the row and column of a_{ij}:

$$A := \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \ldots & a_{mn} \end{bmatrix}$$

Let's start small, where geometric reasoning will help us understand what's going on when we look for solutions to linear equations and to linear systems of equations.

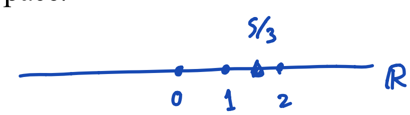
Exercise 2: Describe the solution set of each single linear equation below; describe and sketch its geometric realization in the indicated Euclidean space.

1 eqn, 1 unknown. the real numbers

2a) $3x = 5$, for $x \in \mathbb{R}$.

is an element of

$x = 5/3$



point on the real line

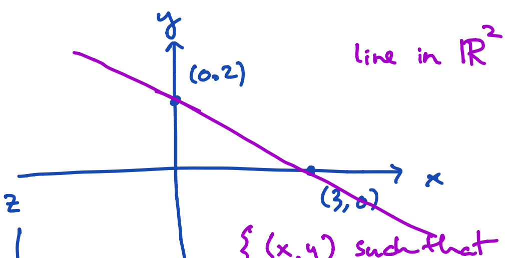
2b) $2x + 3y = 6$, for $[x, y] \in \mathbb{R}^2$.

eg slope intercept form.

$3y = 6 - 2x$

$y = 2 - \frac{2}{3}x$

line in $\mathbb{R}^2$



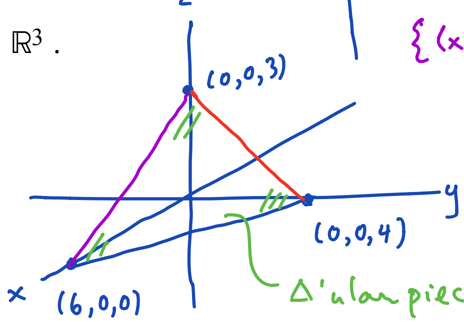
line in $\mathbb{R}^2$

$\{(x,y)$ such that $y = 2 - \frac{2}{3}x\}$

2c) $2x + 3y + 4\boldsymbol{z} = 12$, for $[x, y, \boldsymbol{z}] \in \mathbb{R}^3$.

solution set is a plane

$z = 3 - \frac{1}{2}x - \frac{3}{4}y$



Δ'ular piece of the plane

if $z = 0$, $2x + 3y = 12$

$y = 0$, $2x + 4z = 12$

$x = 0$, $3y + 4z = 12$

2 linear equations in 2 unknowns:

$$a_{11}x + a_{12}y = b_1$$
$$a_{21}x + a_{22}y = b_2$$

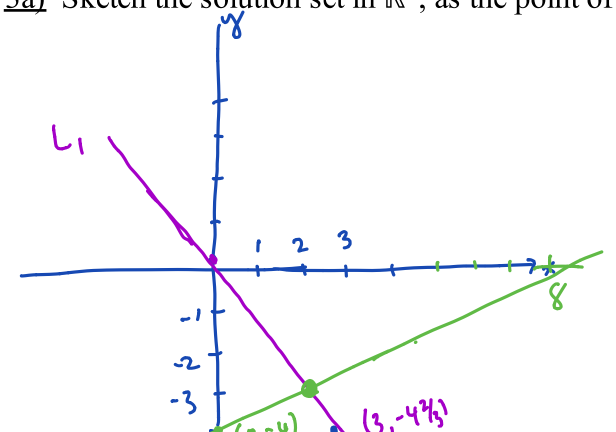
goal: find all $[x, y]$ making both of these equations true at once. Since the solution set to each single equation is a line one geometric interpretation is that you are looking for the intersection of two lines.

Exercise 3: Consider the system of two equations E_1, E_2:

$15 + 3y = 1$
$3y = -14$

$$E_1 \qquad 5x + 3y = 1 \leftarrow (0, \tfrac{1}{3}),\quad (3, -4\tfrac{2}{3})$$
$$E_2 \qquad x - 2y = 8 \quad (8, 0)$$

3a) Sketch the solution set in $\mathbb{R}^2$, as the point of intersection between two lines.



3b) Use the following three "elementary equation operations" to systematically reduce the system E_1, E_2 to an equivalent system (i.e. one that has the same solution set), but of the form

$$1x + 0y = c_1$$
$$0x + 1y = c_2$$

(so that the solution is $x = c_1, y = c_2$). Make sketches of the intersecting lines, at each stage.

The three types of elementary equation operation are below. Can you explain why the solution set to the modified system is the same as the solution set before you make the modification?

- interchange the order of the equations
- multiply one of the equations by a non-zero constant
- replace an equation with its sum with a multiple of a different equation.

$\boxed{\text{HW} \quad 1.1 \ \# \ 1, 3, 13, 17}$