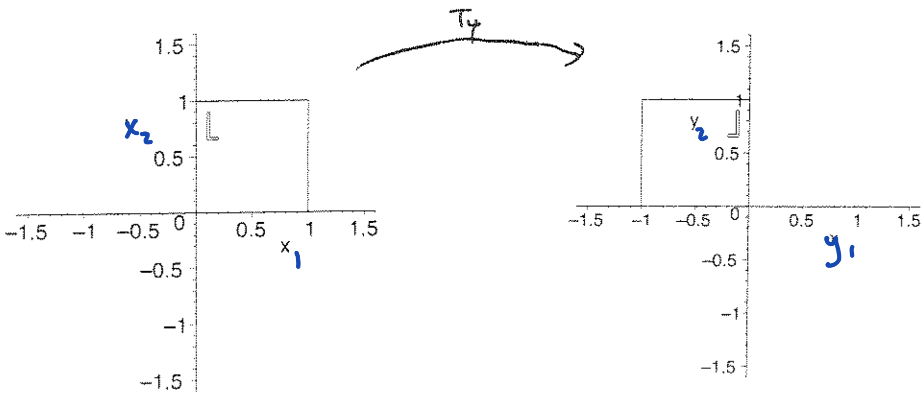
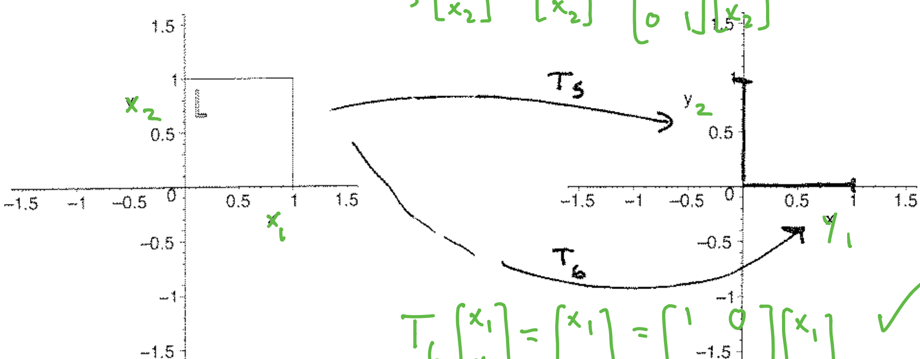


reflect across y-axis

warmup.

$\vec{e}_1 = \begin{bmatrix}1\\0\end{bmatrix}$ position vector $(1,0)$
project $(1,0)$ onto vertical axis I get $(0,0)$

$$T_5\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\x_2\end{bmatrix} = \begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$ ✓



project to y_1-axis

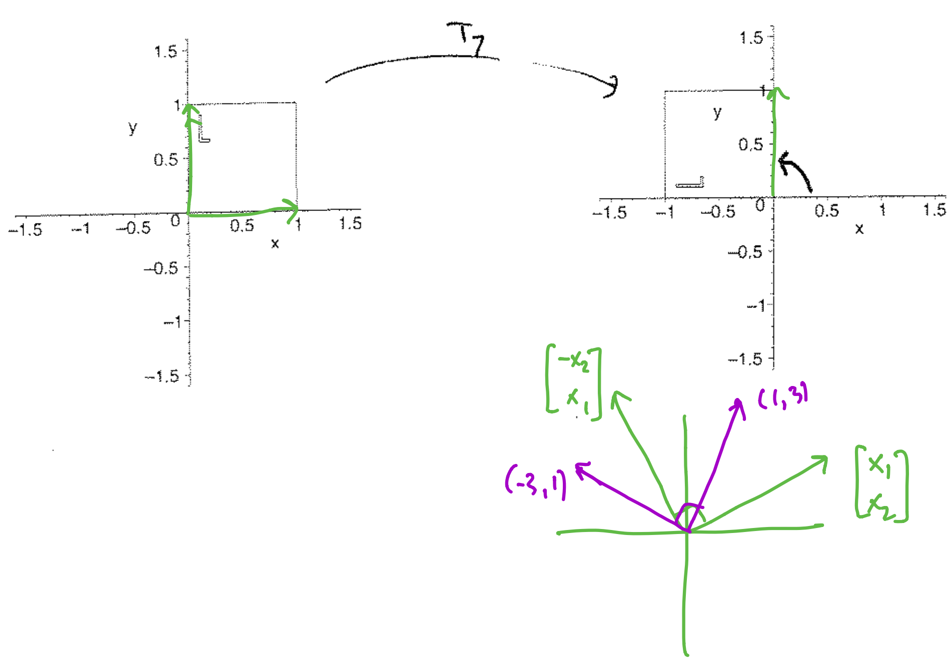
$$A = \left[\, T_5(\vec{e}_1) \;\vdots\; T_5(\vec{e}_2) \,\right] = \begin{bmatrix}0&0\\0&1\end{bmatrix}.$$

$$T_6\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_1\\0\end{bmatrix} = \begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$ ✓

project to x-axis

$$T_6\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix},\quad T_6\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$A = \begin{bmatrix}1&0\\0&0\end{bmatrix}$$



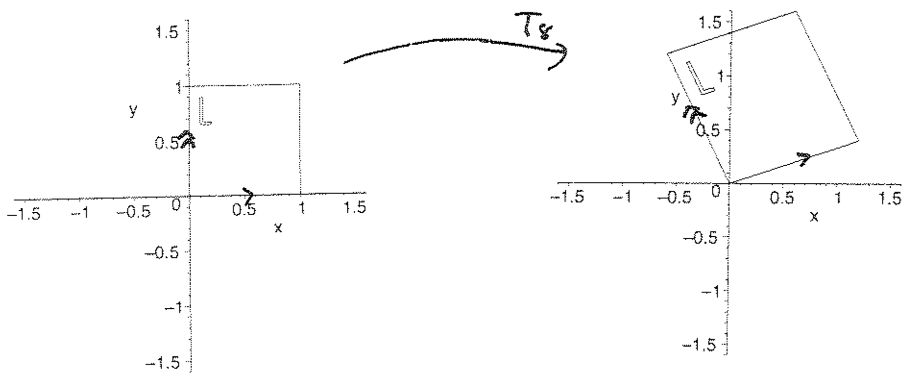
rotate by $\pi/2$ radians counterclockwise

$$T_7\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} \qquad T_7\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$$

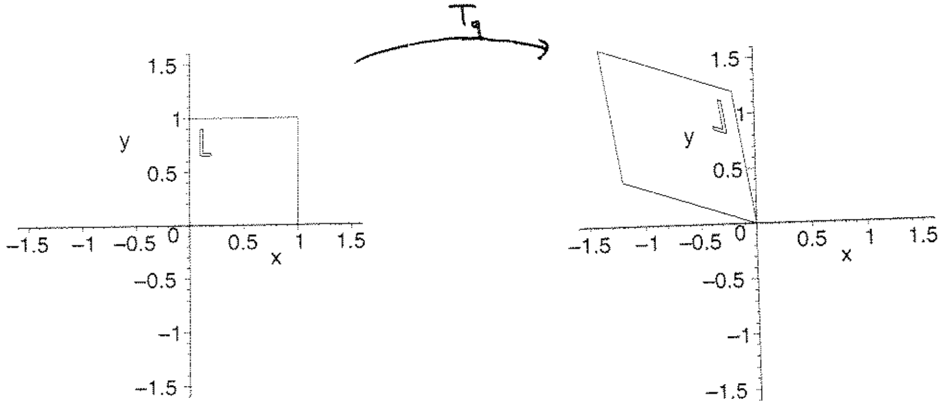
$$A = \begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

$$T_7\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-x_2\\x_1\end{bmatrix}$$

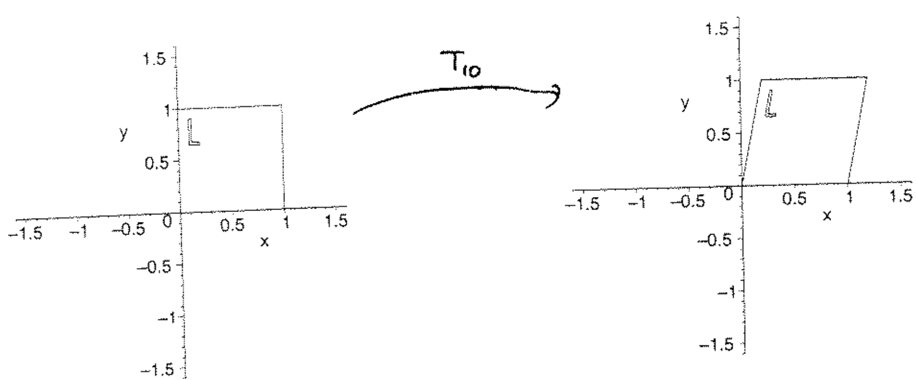
$$\begin{bmatrix}x_1\\x_2\end{bmatrix}\cdot\begin{bmatrix}-x_2\\x_1\end{bmatrix} = -x_1x_2 + x_2x_1 = 0$$



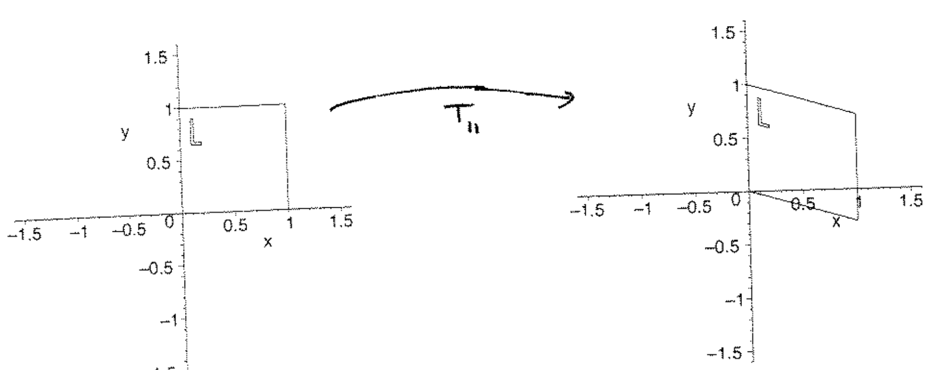
mystery linear trans.



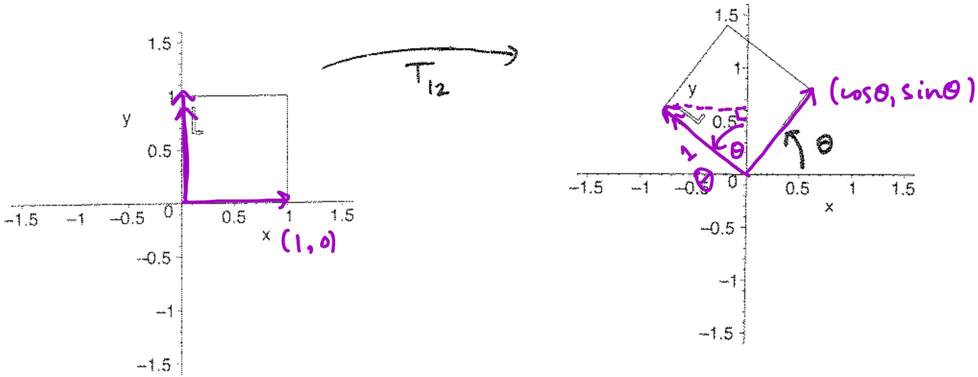
another mystery!



horizontal shear with strength .2



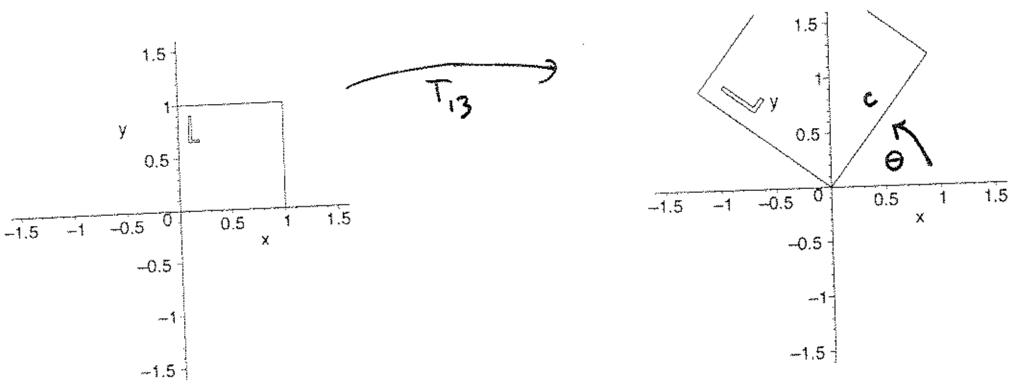
vertical shear with strength -.3



rotation (c.c.) by angle θ

$T_{12}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$ $T_{12}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix}$

$$A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$



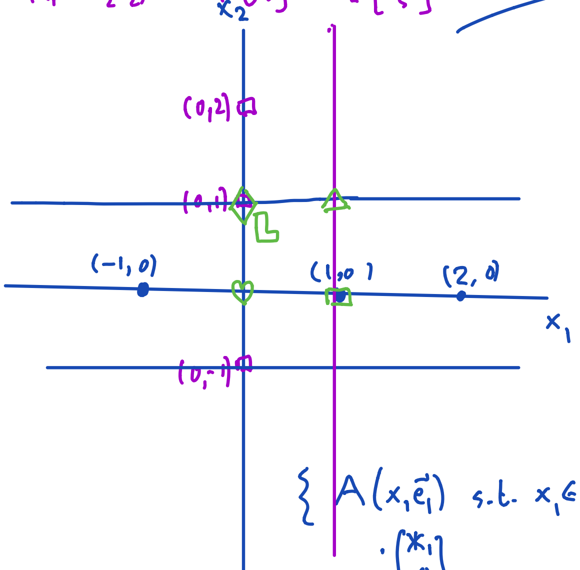
rotate by θ and scale uniformly by factor of c ("rotation dilation")

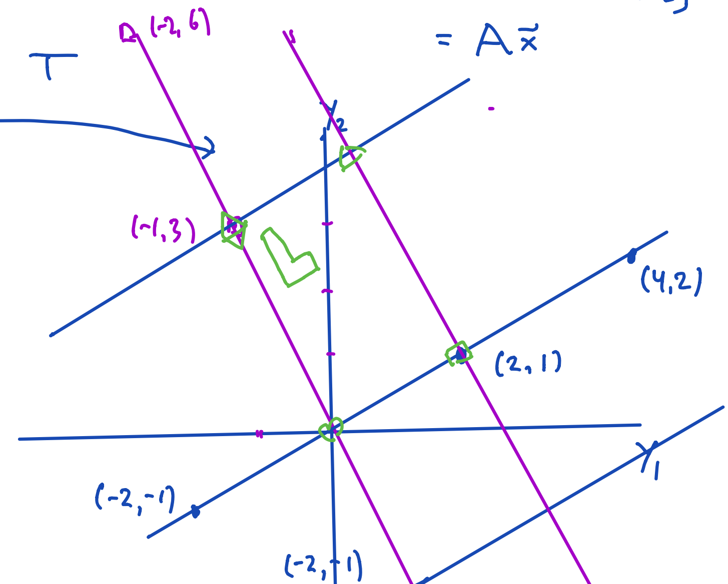
$T: \mathbb{R}^2 \to \mathbb{R}^2$ linear. $\boxed{T(x_1\vec{e}_1 + x_2\vec{e}_2) = x_1 T(\vec{e}_1) + x_2 T(\vec{e}_2)} = \left[T(\vec{e}_1) \,\vdots\, T(\vec{e}_2)\right]\begin{bmatrix}x_1\\x_2\end{bmatrix}$

$= A\vec{x}$

- $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = x_1\begin{bmatrix}2\\1\end{bmatrix} + x_2\begin{bmatrix}-1\\3\end{bmatrix}$

$T(x_1\vec{e}_1 + x_2\vec{e}_2) = x_1\begin{bmatrix}2\\1\end{bmatrix} + x_2\begin{bmatrix}-1\\3\end{bmatrix}$





$\{A(x_1\vec{e}_1) \text{ s.t. } x_1 \in \mathbb{R}\} = \{x_1\begin{bmatrix}2\\1\end{bmatrix}, x_1 \in \mathbb{R}\}$

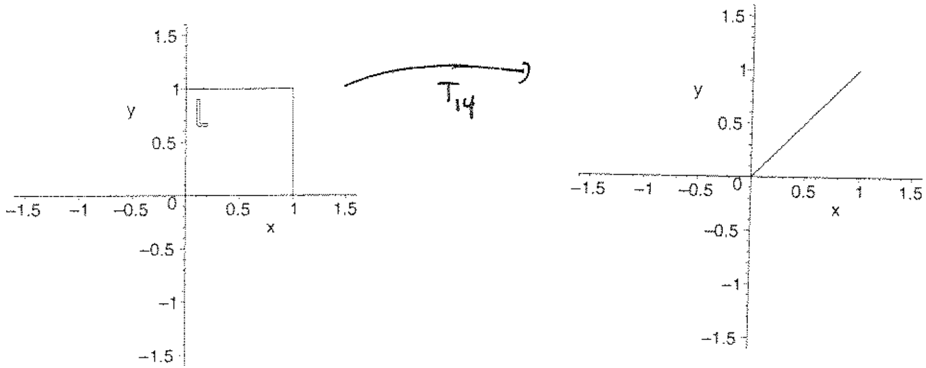
$\begin{bmatrix}x_1\\0\end{bmatrix}$

$\{A(x_2\vec{e}_2) \text{ s.t. } x_2 \in \mathbb{R}\} = \{x_2\begin{bmatrix}-1\\3\end{bmatrix}, x_2 \in \mathbb{R}\}$

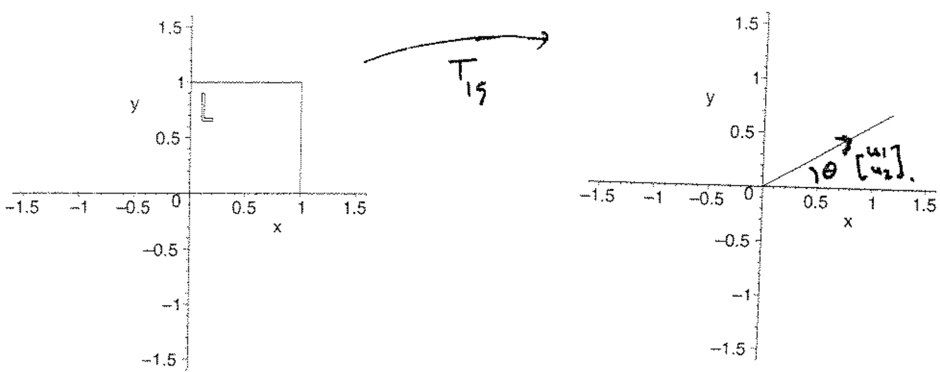
$\begin{bmatrix}0\\x_2\end{bmatrix}$

$$\left\{ A(x_1\vec{e}_1 + 1\cdot\vec{e}_2) = x_1 A(\vec{e}_1) + A(\vec{e}_2) = x_1\begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}-1\\3\end{bmatrix} \right\}$$

$$\left\{ A(\vec{e}_1 + x_2\vec{e}_2) = A\vec{e}_1 + x_2 A\vec{e}_2 = \begin{bmatrix}2\\1\end{bmatrix} + x_2\begin{bmatrix}-1\\3\end{bmatrix} \right\}$$



project onto the line $y=x$



project onto line thru origin at angle θ, with unit direction

$$\begin{bmatrix}u_1\\u_2\end{bmatrix} = \begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix}.$$

Tues Jan 30

- 1.9 the matrix of a linear transformation
- 2.1 Matrix operations, especially matrix multiplication

postpone all 2.1 HW until next assignment

Announcements:

- 2.1 HW: do 1, 3, 9, 11. postpone 23, 25, 27 until next week
- discuss "one to one" and "onto" today.

function concepts: $f: X \longrightarrow Y$ (domain X, codomain Y)

f is "1-1" means the equation $f(x) = b$, $x \in X$, $b \in Y$, has at most one solution $x \in X$.
(if $f(x) = f(z)$, then actually $x = z$)

f is "onto" Y means for each $b \in Y$ there is (at least one) $x \in X$ with $f(x) = b$

HW questions?

~~Warm-up Exercise~~:

If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, given by
$$T(\vec{x}) = A\vec{x} \qquad (A_{m \times n})$$

- then, T being 1-1 means $A\vec{x} = \vec{b}$ has at most one solution. $\Longleftrightarrow$ rref(A) has pivot in each column (i.e. no free parameters).
- then T being onto means $A\vec{x} = \vec{b}$ has (at least) a solution $\vec{x}$ for each $\vec{b} \in \mathbb{R}^m$ $\Longleftrightarrow$ rref(A) has no zero rows, i.e. a pivot in each row.

- continue understanding the geometry of linear transformations, using Monday's notes